محمدمهدی غفاریان / دوازدهم پسرانه A

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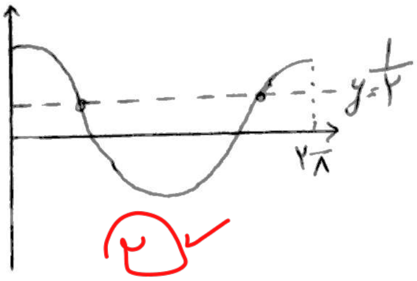
$8\cos x = 1+\tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x}$

$\to 8\cos^3 x = 1 \to \cos^3 x = \frac{1}{8}$

$\to \cos x = \frac{1}{2} = \cos\frac{\pi}{3} \to x = 2k\pi \pm \frac{\pi}{3}$



۲ جواب

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$\sin^2 x = \frac{1-\cos 2x}{2} \Rightarrow 5\sin^2 x = \frac{5}{2}(1-\cos 2x)$

$\frac{5}{2}(1-\cos 2x)+2\cos 3x=-2$

$\xrightarrow{\times 2} 5(1-\cos 2x)+4\cos 3x=-4$

$\to 5\underbrace{\cos 2x}_{2\cos^2 x-1}-4\underbrace{\cos 3x}_{4\cos^3 x-3\cos x}=9$

$\to 10\cos^2 x-5-16\cos^3 x+12\cos x=9$

$\to 16\cos^3 x-10\cos^2 x-12\cos x+14=0$

$t=\cos x$ و $-1\le t\le 1$

$16t^3-10t^2-12t+14=0 \xrightarrow{t=-1}$

$(t+1)(\underbrace{16t^2-26t+14}_{\Delta<0})=0$

$\Rightarrow t=\cos x=-1 \Rightarrow x=2k\pi+\pi \to$ ۱ جواب

$-\pi$ و π

1,75

-۳ 14,75

$\sin x\cdot(2\underbrace{\cos 2x}_{1-2\sin^2 x}+1)=1$

$\to(\sin x)(3-4\sin^2 x)=1$

$\sin x=t \to t(3-4t^2)=1 \to -4t^3+3t-1=0$

$\xrightarrow{t=-1}(t+1)(4t^2-4t+1)=0$

$t=-1$ ، $t=\frac{1}{2}$

① $\sin x=-1 \to x=\frac{3\pi}{2}$

② $\sin x=\frac{1}{2} \to x=\frac{\pi}{6}$ و $x=\frac{5\pi}{6}$

مجموع جواب‌ها $=\frac{3\pi}{2}+\frac{\pi}{6}+\frac{5\pi}{6}=\frac{5\pi}{2}$

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$\frac{1}{\sin(\frac{\pi}{2}+2x)}+\frac{1}{\cos(\frac{\pi}{2}+4x)}=0$

$\to \frac{1}{\cos 2x}-\frac{1}{\sin 4x}=0 \Rightarrow \frac{1}{\cos 2x}=\frac{1}{\sin 4x}$

$\to \cos 2x=\underbrace{\sin 4x}_{2\sin 2x\cos 2x} \xrightarrow{\div\cos 2x} 2\sin 2x=1$

$\to \sin 2x=\frac{1}{2}$ $\begin{cases} 2x=2k\pi+\frac{\pi}{6} \to x=k\pi+\frac{\pi}{12} \\ 2x=2k\pi+\frac{5\pi}{6} \to x=k\pi+\frac{5\pi}{12} \end{cases}$

$\to \alpha=\frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3} \to \tan 2\alpha=\tan\frac{2\pi}{3}=-\sqrt{3}$

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$\cos(x+\frac{\pi}{4})=\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}$

$\xrightarrow{\cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}} \frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}}$

$\xrightarrow{\times\frac{2}{\sqrt{2}}} \cos x-\sin x=\frac{2}{\sqrt{6}}$

$\underbrace{(\cos x-\sin x)^2}_{2/\sqrt{6}}=1-\sin 2x \to \frac{2}{3}=\underbrace{1-\sin 2x}_{\to 1/3}$

$\to m(\frac{2}{\sqrt{6}})-\sqrt{6}=\sqrt{6} \Rightarrow m=6$

۲ ✓

۹- $(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$

$\times 8\sin^2\alpha \rightarrow \left(\frac{1}{2}\sin^2 2\alpha\right)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}\sin^2\alpha$

$\left(\frac{1}{4}\sin^2 4\alpha\right)(2\cos^2 4\alpha) = \frac{1}{8}\sin^2\alpha$

$\Rightarrow \frac{1}{8}\sin^2 8\alpha = \frac{1}{8}\sin^2\alpha \Rightarrow 8\alpha = k\pi \pm \alpha$

$\Rightarrow \begin{cases} 7\alpha = k\pi \Rightarrow \alpha = \frac{k\pi}{7} \\ 9\alpha = k\pi \Rightarrow \alpha = \frac{k\pi}{9} \end{cases}$

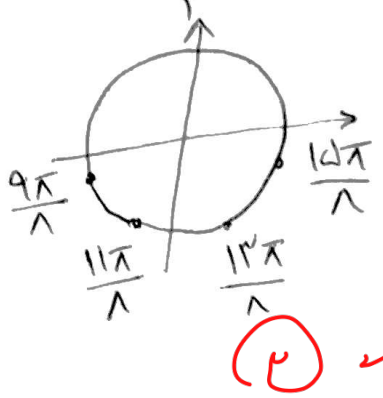
$A = \left\{\frac{\pi}{9}, \frac{\pi}{7}, \frac{2\pi}{9}, \frac{2\pi}{7}, \dots \frac{8\pi}{9}\right\}$

$\max = \frac{8\pi}{9}$ ✓

۱۰- $\tan 2x \cdot \tan x = 1 \Rightarrow \tan 2x = \cot x$

$\Rightarrow \tan 2x = \tan\left(\frac{\pi}{2} - x\right)$

$\Rightarrow 2x = \frac{\pi}{2} - x + k\pi \Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{6}$



$S = \frac{9\pi + 11\pi + 13\pi + 15\pi}{8}$

$= \frac{48\pi}{8} = 6\pi$ ✓

۶- $x - \frac{\pi}{2} = \alpha \rightarrow x + \frac{\pi}{4} = \alpha + \frac{\pi}{2}$

$\rightarrow \sin\left(\frac{\pi}{2} + \alpha\right) \cdot \cos\alpha = 1 \rightarrow \cos^2\alpha = 1$

$\rightarrow \alpha = k\pi \rightarrow x - \frac{\pi}{2} = k\pi \rightarrow x = k\pi + \frac{\pi}{2}$ ✓

۷- $\frac{1}{111}\sqrt{1^2 + (\sqrt{3})^2}\sin(x + \alpha)$

$\hookrightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\rightarrow \sin x + \sqrt{3}\cos x = 2\sin\left(x + \frac{\pi}{3}\right) = \sqrt{2}$

$\rightarrow \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$

$\hookrightarrow \begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12} \end{cases}$

$k=0 \begin{cases} x = -\frac{\pi}{12} \checkmark \\ x = \frac{5\pi}{12} \checkmark \end{cases}$ $k=1 \begin{cases} x = \frac{23\pi}{12} \checkmark \\ x = \frac{29\pi}{12} \rightarrow \text{بیش} \end{cases}$

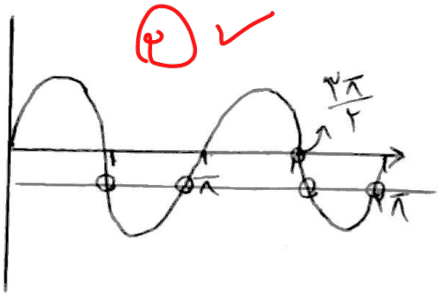
$k=-1 \begin{cases} x = -\frac{7\pi}{12} \\ x = -\frac{19\pi}{12} \end{cases}$ بیش ✓

$S = \frac{23\pi + 5\pi - \pi}{12} = \frac{27\pi}{12}$

۸- $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

$\rightarrow 4\sin x\cos x = -1 \rightarrow 2\sin 2x = -1$

$\rightarrow \sin 2x = -\frac{1}{2}$ ✓



$\frac{\frac{3\pi}{2} + \pi}{2} = \frac{5\pi}{4}$

$\hookrightarrow 4 \times \frac{5\pi}{4} = 5\pi$