متین شهبازی

۱۴/۱۵

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$8\cos x - \tan^2 x = 1 \quad [0, 2\pi]$

$8\cos x = 1 + \tan^2 x \Rightarrow \frac{8\cos x}{1} = \frac{1}{\cos^2 x}$ $\quad 8\cos^3 x = 1 \quad \cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \quad x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\} \Longrightarrow$ پاسخ ۳

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$5\sin^2 x + 2\cos 3x = -3$

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -3$ $\quad 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -3$

$\cos^2 \frac{3x}{2} = \frac{1+\cos 3x}{2}$

$5\sin^2 x + 2\left(2\cos^2 \frac{3x}{2} - 1\right) = -3$

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$5\sin^2 x + 4\cos^2 \frac{3x}{2} = 0$

$x = \pi, 0, -\pi \qquad x = -\pi, \frac{\pi}{3}, -\frac{\pi}{3}, \pi$

$\{-\pi, \pi\}$

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$2\sin x \cos(2x) + \sin x = 1 \quad , [0, 2\pi]$

~~$2\sin x(2\cos^2 x - 1) + \sin x = 1$~~ ~~$4\cos^2 x \sin x - \sin x = 1$~~ ~~$\sin x(4\cos^2 x - 1) = 1$~~

~~$4\cos^2 x \sin x - 2\sin x + \sin x = 1$~~

$2\sin x(\underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x) + \sin x = 1 \qquad 2\sin x - 4\sin^3 x + \sin x = 1 \qquad 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \qquad 3x = 2k\pi + \frac{\pi}{2} \qquad x = \frac{2k\pi}{3} + \frac{\pi}{6} \qquad x = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}\right\}$

$\boxed{\frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6}}$

۲ ✓

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0 \qquad \frac{1}{\cos 2x} - \frac{1}{-\sin 4x} = 0 \qquad \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} = \phi$$

$$\frac{2\sin 2x\cos 2x - \cos 2x}{\cos 2x \sin 4x} = 0 \qquad \frac{\cos 2x\,(2\sin 2x - 1)}{\cos 2x \sin 4x} = 0 \qquad 2\sin 2x = 1 \quad \sin 2x = \frac{1}{2} \qquad 2x = \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} \quad x = \left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$$

$$\alpha = \frac{\pi}{3} \qquad \boxed{\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}} \qquad \Delta x = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$$

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-۵

$$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \ (2) \qquad \cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \ (1)$$

$$(1)\quad \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x = \frac{1}{\sqrt{2}} \qquad \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{2}} \qquad (\cos x - \sin x) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1}$$

$$\cos x - \sin x = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{3} \qquad (\cos x - \sin x)^2 = \left(\frac{\sqrt{6}}{3}\right)^2 \to \sin 2x = \frac{1}{3}$$ ✓

$$(2)\quad m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \qquad m\left(\frac{\sqrt{6}}{3}\right) - 3\sqrt{6}\sin 2x = \sqrt{6} \qquad \frac{m}{3} - 3\cdot\cancel{\sin 2x}^{\frac{1}{3}} = 1$$

۱٫۲۵

$$\cancel{-9\sin 2x = 3 - m \quad \sin 2x = \frac{m-3}{9} \quad -1 \le \frac{m-3}{9} \le 1 \quad -9 \le m-3 \le 9} \quad -6 \le m \le 12 \quad m \in [-6, 12]$$

$$\frac{m}{3} - 1 = 1 \qquad m = 6$$

-۶

$$\sin\left(x + \frac{\pi}{4}\right)\cos\left(x - \frac{\pi}{4}\right) = 1 \qquad [0, 2\pi]$$

$$\cos x = \cos(-x) \qquad \sin^2\left(x + \frac{\pi}{4}\right) = 1 \qquad \sin\left(x + \frac{\pi}{4}\right) = \pm 1$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4} - x\right) = \sin\left(x + \frac{\pi}{4}\right) \qquad x + \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{2}$$

$$x = 2k\pi + \frac{\pi}{2} - \frac{\pi}{4} \qquad \left\{\cancel{\frac{2\pi}{4}}\right\} \quad \frac{\pi}{4}$$

$$x = 2k\pi - \frac{\pi}{2} - \frac{\pi}{4} \qquad \left\{\cancel{\frac{10\pi}{4}}\right\} \quad \frac{5\pi}{4}$$

$\boxed{\text{۲ جواب}}$ ۱٫۵ ✓

-۷

$$\sin x + \sqrt{3}\cos x = \sqrt{2} \qquad [-\pi, 2\pi]$$

$$2\sin\left(x + \frac{\pi}{3}\right) = \sqrt{2} \qquad \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

۲ ✓

$$\boxed{-\frac{\pi}{12} + \frac{23\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \frac{9\pi}{4}}$$

$$x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \qquad x = 2k\pi + \frac{\pi}{4} - \frac{\pi}{3} \Rightarrow x = 2k\pi - \frac{\pi}{12} \qquad x = \left\{-\frac{\pi}{12}, \frac{23\pi}{12}\right\}$$

$$x + \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{4} \qquad x = 2k\pi + \pi - \frac{\pi}{4} - \frac{\pi}{3} \Rightarrow x = 2k\pi + \pi - \frac{7\pi}{12} \qquad x = \left\{\frac{5\pi}{12}\right\}$$

-۸

$۴\sin x \sin(\frac{۳\pi}{۲}-x)=۱ \quad -۲(۲\sin x\cos x)=۱ \quad \sin ۲x=-\frac{۱}{۲} \quad ۲x=۲k\pi-\frac{\pi}{۶} , ۲x=۲k\pi+\pi+\frac{\pi}{۶}$

$x=k\pi-\frac{\pi}{۱۲} \quad x=\{\frac{۱۱\pi}{۱۲},\frac{۲۳\pi}{۱۲}\}$

$x=k\pi+\frac{\pi}{۲}+\frac{\pi}{۱۲} \quad x=\{\frac{۷\pi}{۱۲},\frac{۱۹\pi}{۱۲}\}$

$\boxed{\frac{۱۱\pi}{۱۲}+\frac{۲۳\pi}{۱۲}+\frac{۷\pi}{۱۲}+\frac{۱۹\pi}{۱۲}=\frac{۶۰\pi}{۱۲}}=۵\pi$

۱,۲۵

-۹

$(1+\cos(۲\alpha))(1+\cos(۴\alpha))(1+\cos(۸\alpha))=\frac{۱}{۸}$

$\cancel{\frac{۱}{۴}\sin^۲ ۴\alpha} \quad \frac{۱}{۴}\sin^۲ ۸\alpha$

$(۲\cos^۲\alpha)(۲\cos^۲ ۲\alpha)(۲\cos^۲ ۴\alpha)\times\frac{\sin^۲\alpha}{\sin^۲\alpha}=\frac{\frac{۱}{۴}(\sin^۲ ۲\alpha\times ۲\cos^۲ ۲\alpha)\times ۲\cos^۲ ۴\alpha}{\sin^۲\alpha}=\cancel{\frac{۱}{۸}}\frac{\sin^۲ ۸\alpha}{\sin^۲\alpha}=\cancel{\frac{۱}{۸}}$

$\frac{\sin^۲ ۸\alpha}{\sin^۲\alpha}=۱ \quad \frac{\sin ۸\alpha}{\sin\alpha}=\pm۱ \quad \sin ۸\alpha=\pm\sin\alpha \quad ۸\alpha=k\pi\pm\alpha$

$\max A=\frac{۸\pi}{۹}$

$۸\alpha=k\pi+\alpha \quad ۷\alpha=k\pi \quad \alpha=\frac{k\pi}{۷} \quad \frac{۲k\pi+\pi}{۷}$

$۸\alpha=k\pi-\alpha \quad ۹\alpha=k\pi \quad \alpha=\frac{k\pi}{۹} \quad \boxed{k_{max}=۹} \quad \frac{۲k\pi+\pi}{۹}$

۱

$۸\alpha=k\pi+\pi-\alpha$

$۸\alpha=k\pi+(\pi\pm\alpha)$

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$, [\pi,۲\pi]$

$\tan(۳x)\tan(x)=۱ \quad \tan ۳x=\cot x \quad \tan ۳x=\tan(\frac{\pi}{۲}-x)$

$۳x=k\pi+\frac{\pi}{۲}-x \quad ۴x=k\pi+\frac{\pi}{۲} \quad x=\frac{k\pi}{۴}+\frac{\pi}{۸} \quad \boxed{x=[\cancel{\frac{۹\pi}{۸}}]}$

K	۴	۵	۶	۷
x	$\frac{۹\pi}{۸}+$	$\frac{۱۱\pi}{۸}+$	$\frac{۱۳\pi}{۸}+$	$\frac{۱۵\pi}{۸}$

$=۶\pi$

۱