حسین شیرازی

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$8\cos x - \tan^2 x = 1 \quad [0, 2\pi]$

$8\cos^3 x = 1 \qquad \cos^3 x = \frac{1}{8}$

$8\cos x = 1 + \tan^2 x \Rightarrow \frac{8\cos x}{1} = \frac{1}{\cos^2 x} \qquad \cos x = \frac{1}{2} \qquad x = \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ $\Longrightarrow$ ۲ پاسخ

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$5\sin^2 x + 2\cos 3x = -2$

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2 \qquad 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2$

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$2\sin x \times \cos(2x) + \sin x = 1 \qquad , [0, 2\pi]$

$\cancel{2\sin x(2\cos^2 x - 1) + \sin x = 1} \qquad \cancel{4\cos^2 x \sin x - \sin x = 1} \qquad \cancel{\sin x(4\cos^2 x - 1) = 1}$

$\cancel{4\cos^2 x \sin x - 2\sin x + \sin x = 1}$

$2\sin x(\underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x) + \sin x = 1 \qquad 2\sin x - 4\sin^3 x + \sin x = 1 \qquad 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \qquad 3x = 2k\pi + \frac{\pi}{2} \qquad x = \frac{2k\pi}{3} + \frac{\pi}{6} \qquad x = \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}\right\}$

$\boxed{\frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6}}$

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)}+\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}=0 \qquad \frac{1}{\cos 2x}-\frac{1}{\sin 4x}=0 \qquad \frac{\sin 4x-\cos 2x}{\cos 2x\,\sin 4x}=0$$

$$\frac{2\sin 2x\cos 2x-\cos 2x}{\cos 2x\,\sin 4x}=0 \qquad \frac{\cos 2x\,(2\sin 2x-1)}{\cos 2x\,\sin 4x}=0 \qquad 2\sin 2x=1 \quad \sin 2x=\frac{1}{2} \qquad 2x=\left\{\frac{\pi}{6},\frac{5\pi}{6}\right\} \quad x=\left\{\frac{\pi}{12},\frac{5\pi}{12}\right\}$$

$$\alpha=\frac{\pi}{3} \qquad \boxed{\tan 2\alpha=\tan\frac{2\pi}{3}=-\sqrt{3}} \qquad \Delta x=\frac{5\pi}{12}-\frac{\pi}{12}=\frac{4\pi}{12}=\frac{\pi}{3}$$

-۵

$$\underset{(2)}{m(\cos x-\sin x)-3\sqrt{6}\sin 2x=\sqrt{6}} \quad , \quad \underset{(1)}{\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}}$$

$$(1)\quad \cos\frac{\pi}{4}\cos x-\sin\frac{\pi}{4}\sin x=\frac{1}{\sqrt{3}} \qquad \frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}} \qquad (\cos x-\sin x)=\frac{1}{\sqrt{3}}\times\frac{2}{\sqrt{2}}$$

$$\cos x-\sin x=\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$$

$$(2)\quad m(\cos x-\sin x)-3\sqrt{6}\sin 2x=\sqrt{6} \qquad m\left(\frac{\sqrt{6}}{3}\right)-3\sqrt{6}\sin 2x=\sqrt{6} \qquad \frac{m}{3}-3\sin 2x=1$$

$$-9\sin 2x=3-m \qquad \sin 2x=\frac{m-3}{9} \qquad -1\leq\frac{m-3}{9}\leq 1 \quad -9\leq m-3\leq 9 \quad -6\leq m\leq 12 \qquad m\in[-6,12]$$

-۶

$$\sin\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right)=1 \quad , [0,2\pi]$$

$$\cos x=\cos(-x) \qquad \sin^2\left(x+\frac{\pi}{4}\right)=1 \quad \sin\left(x+\frac{\pi}{4}\right)=\pm 1$$

$$\cos\left(x-\frac{\pi}{4}\right)=\cos\left(\frac{\pi}{4}-x\right)=\sin\left(x+\frac{\pi}{4}\right) \qquad x+\frac{\pi}{4}=2k\pi\pm\frac{\pi}{2}$$

$$x=2k\pi+\frac{\pi}{2}+\frac{\pi}{4} \qquad \left\{\frac{3\pi}{4}\right\}$$

$$x=2k\pi-\frac{\pi}{2}+\frac{\pi}{4} \qquad \left\{\frac{10\pi}{4}\right\}$$

$\boxed{\text{۲ جواب}}$

-۷

$$\sin x+\sqrt{3}\cos x=\sqrt{2} \quad , \quad [-\pi,2\pi]$$

$$2\sin\left(x+\frac{\pi}{3}\right)=\sqrt{2} \qquad \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2} \qquad \boxed{-\frac{\pi}{12}+\frac{23\pi}{12}+\frac{5\pi}{12}=\frac{27\pi}{12}=\frac{9\pi}{4}}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \qquad x=2k\pi+\frac{\pi}{4}-\frac{\pi}{3} \Rightarrow x=2k\pi-\frac{\pi}{12} \qquad x=\left\{\frac{-\pi}{12},\frac{23\pi}{12}\right\}$$

$$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4} \qquad x=2k\pi+\pi-\frac{\pi}{4}-\frac{\pi}{3} \Rightarrow x=2k\pi+\pi-\frac{7\pi}{12} \qquad x=\left\{\frac{5\pi}{12}\right\}$$

-۸

$۴\sin x \sin\left(\frac{۳\pi}{۲} - x\right) = ۱ \quad -۲(۲\sin x \cos x) = ۱ \quad \sin ۲x = -\frac{۱}{۲} \quad ۲x = ۲k\pi - \frac{\pi}{۶} , ۲x = ۲k\pi + \pi + \frac{\pi}{۶}$

$x = k\pi - \frac{\pi}{۱۲} \quad x = \left\{\frac{۱۱\pi}{۱۲}, \frac{۲۳\pi}{۱۲}\right\}$

$\boxed{\frac{۱۱\pi}{۱۲} + \frac{۲۳\pi}{۱۲} + \frac{۷\pi}{۱۲} + \frac{۱۹\pi}{۱۲} = \frac{۶۰\pi}{۱۲}}$

$x = k\pi + \frac{\pi}{۲} + \frac{\pi}{۱۲} \quad x = \left\{\frac{۷\pi}{۱۲}, \frac{۱۹\pi}{۱۲}\right\}$

-۹

$(۱+\cos(۲\alpha))(۱+\cos(۴\alpha))(۱+\cos(۸\alpha)) = \frac{۱}{۸}$

$\frac{۱}{۴}\sin^۲ ۴\alpha \qquad \frac{۱}{۱۶}\sin^۲ ۸\alpha$

$(۲\cos^۲\alpha)(۲\cos^۲ ۲\alpha)(۲\cos^۲ ۴\alpha) \times \frac{\sin^۲\alpha}{\sin^۲\alpha} = \frac{\frac{۱}{۴}(\sin^۲ ۲\alpha \times ۲\cos^۲ ۲\alpha) \times ۲\cos^۲ ۴\alpha}{\sin^۲\alpha} = \cancel{\frac{۱}{۸}} \frac{\sin^۲ ۸\alpha}{\sin^۲\alpha} = \cancel{\frac{۱}{۸}}$

$\frac{\sin^۲ ۸\alpha}{\sin^۲\alpha} = ۱ \quad \frac{\sin ۸\alpha}{\sin\alpha} = \pm ۱ \quad \sin ۸\alpha = \pm \sin\alpha \quad ۸\alpha = k\pi \pm \alpha$

$۸\alpha = k\pi + \alpha \quad ۷\alpha = k\pi \quad \alpha = \frac{k\pi}{۷}$

$۸\alpha = k\pi - \alpha \quad ۹\alpha = k\pi \quad \alpha = \frac{k\pi}{۹} \quad \boxed{k_{max} = ۹}$

-۱۰

$\in [\pi, ۲\pi]$

$\tan(۳x)\tan(x) = ۱ \quad \tan ۳x = \cot x \quad \tan ۳x = \tan\left(\frac{\pi}{۲} - x\right)$

$۳x = k\pi + \frac{\pi}{۲} - x \quad ۴x = k\pi + \frac{\pi}{۲} \quad x = \frac{k\pi}{۴} + \frac{\pi}{۸} \quad \boxed{x = \left[\frac{۹\pi}{۸}\right]}$