دوازدهم ریاضی A — تکلیف شماره ۱۷ — ۱۸

۱- $8\cos\alpha - \tan^2\alpha = 1 \Rightarrow 8\cos\alpha = 1+\tan^2\alpha \Rightarrow 8\cos\alpha = \frac{1}{\cos^2\alpha}$

$\Rightarrow 8\cos^3\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow$ جواب ها $\Rightarrow \alpha = \frac{\pi}{3}, \frac{5\pi}{3}$

۲- $5\sin^2\alpha + 4\cos 3\alpha = -2 \Rightarrow 5 - 5\cos^2\alpha + 16\cos^3\alpha - 12\cos\alpha + 2 = 0$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha \qquad 16\cos^3\alpha - 5\cos^2\alpha - 12\cos\alpha + 7 = 0$

$(\cos\alpha + 1)(16\cos^2\alpha - 21\cos\alpha + 7) = 0 \Rightarrow \cos\alpha = -1 \Rightarrow \alpha = \pi, -\pi$

$\Delta < 0$

جواب ندارد

۳- $2\sin\alpha\cos 2\alpha + \sin\alpha = 1 \Rightarrow ~~2\sin\alpha~~ - 4\sin^3\alpha + ~~\sin\alpha~~ = 1$

$3\sin\alpha$

$1 - 2\sin^2\alpha \qquad \Rightarrow (\sin\alpha + 1)(2\sin\alpha - 1)^2 = 0 \qquad \sin\alpha = -1, \ \sin\alpha = \frac{1}{2}$

$\sin\alpha = -1 \Rightarrow \alpha = \frac{3\pi}{2}, \ \sin\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow \frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{2}$

۴- $\sin(\frac{\pi}{2} + 2x)^{-1} + \cos(\frac{\pi}{2} + 4x)^{-1} \Rightarrow \cos 2x = \sin 4x$

$\Rightarrow \cos 2x = 2\sin 2x\cos 2x \Rightarrow 1 = 2\sin 2\alpha \Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow$

$2\alpha = 2k\pi + \frac{\pi}{4} \Rightarrow \alpha = k\pi + \frac{\pi}{8}$ أو $2\alpha = 2k\pi - \frac{\pi}{4} \Rightarrow \alpha = k\pi - \frac{\pi}{8}$

$\Rightarrow \alpha = \frac{5\pi}{8}, -\frac{\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}$ ... $\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$

$\cos(\alpha + \frac{\pi}{4}) = \frac{1}{\sqrt{10}} \Rightarrow \cos(\alpha + \frac{\pi}{4})^2 = \frac{1}{10} \Rightarrow -\sin 2\alpha = -\frac{1}{10}$ ... (1)

$\Rightarrow \sin 2\alpha = \frac{1}{10}$ $\quad \cos(2\alpha + \frac{\pi}{2}) = m(-\sqrt{2}\sin(\alpha - \frac{\pi}{4}))$

$\Rightarrow |\sqrt{2}\sin(\frac{\pi}{4} - \alpha)| = m = \frac{\sqrt{6}}{3}$ $m = \sqrt{6}, -\sqrt{6}$

$\Rightarrow$ [illegible] $m = \frac{2\sqrt{6}}{\frac{\sqrt{6}}{3}} \Rightarrow m = 6$

$\sin(\alpha + \frac{\pi}{6})\cos(\alpha - \frac{\pi}{3}) = 1 \Rightarrow \cos(\frac{\pi}{3} - \alpha)\cos(\alpha - \frac{\pi}{3}) = 1$

$\Rightarrow$ [illegible] $\sin^2(\alpha + \frac{\pi}{6}) = 1 \Rightarrow \sin(\alpha + \frac{\pi}{6}) = \pm 1$

$\Rightarrow \alpha + \frac{\pi}{6} = k\pi + \frac{\pi}{2} \Rightarrow \alpha = k\pi + \frac{\pi}{3}$ ... [illegible]

$\sin\alpha + \sqrt{3}\cos\alpha = \sqrt{3}$ $\xrightarrow{\div 2}$ $\sin(\alpha + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ ... (2)

$\Rightarrow \alpha + \frac{\pi}{3} = 2k\pi + \frac{\pi}{3} \Rightarrow \alpha = 2k\pi - \frac{\pi}{3}$ أو $\alpha + \frac{\pi}{3} = 2k\pi + \frac{2\pi}{3}$

$\Rightarrow \alpha = 2k\pi + \frac{\pi}{3} \Rightarrow \alpha = \frac{5\pi}{3}, \frac{\pi}{3}, \frac{11\pi}{3}$

$\frac{5\pi}{3} - \frac{\pi}{3} + \frac{11\pi}{3} = \frac{15\pi}{3}$

$4\sin\alpha\sin\left(\frac{3\pi}{2}-\alpha\right)=1 \Rightarrow 4\sin\alpha(-\cos\alpha)=1 \Rightarrow -4\sin\alpha\cos\alpha=-1$ ... ۸

$\Rightarrow \sin 2\alpha=\frac{1}{2} \Rightarrow 2\alpha=2k\pi+\frac{\pi}{6} \Rightarrow \alpha=k\pi+\frac{\pi}{12}$ یا $2\alpha=2k\pi+\frac{5\pi}{6}$

$\Rightarrow \alpha=k\pi+\frac{5\pi}{12}$ ✓ ۲

$\xrightarrow{\text{مجموع}} \frac{\pi}{12}+\frac{11\pi}{12}+\frac{13\pi}{12}+\frac{23\pi}{12}=4\pi$

$1+\cos 2\alpha=2\cos^2\alpha$

$1+\cos 4\alpha=2\cos^2 2\alpha$ ۹-

$1+\cos 8\alpha=2\cos^2 4\alpha$

$\cos\alpha\cos 4\alpha\cos 2\alpha=\pm\frac{1}{8}$ ۵

$\tan 2\alpha\tan\alpha=1 \Rightarrow \tan 2\alpha=\frac{1}{\tan\alpha} \Rightarrow \tan 2\alpha=\tan\left(\frac{\pi}{2}-\alpha\right)$ ۶-

$\Rightarrow 2\alpha=k\pi+\frac{\pi}{2}-\alpha \xrightarrow{\times 3} 6\alpha=k\pi+\frac{\pi}{2} \Rightarrow \alpha=\frac{k\pi}{6}+\frac{\pi}{12}$

$\xrightarrow{\text{مجموع}} \frac{9\pi}{8}+\frac{17\pi}{8}+\frac{25\pi}{8}+\frac{33\pi}{8}=6\pi$ ✓ ۲

$\frac{\times\sin\alpha}{\sin\alpha\neq 0}$ $\sin 8\alpha=\pm\sin\alpha$

$8\alpha=2k\pi+\alpha$ $8\alpha=2k\pi+\pi-\alpha$

$8\alpha=2k\pi-\alpha$ $8\alpha=2k\pi+\pi+\alpha$

$\alpha=\frac{2k\pi}{9}$

$\alpha=\frac{2k\pi}{7}$

$\alpha=\frac{2k\pi+\pi}{9}$

$\alpha=\frac{2k\pi+\pi}{7}$

$\max(A)=\frac{8\pi}{7}$