بارسا حقیقی — تکلیف شماره ۱۷ — دوازدهم ب A

۱- $8\cos\alpha - \tan^2\alpha = 1 \Rightarrow 8\cos\alpha = 1 + \tan^2\alpha \Rightarrow 8\cos\alpha = \frac{1}{\cos^2\alpha}$

$\Rightarrow 8\cos^3\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow$ ۲ جواب

۲- $5\sin^2\alpha + 2\cos 3\alpha = -2 \Rightarrow 5 - 5\cos^2\alpha + 8\cos^3\alpha - 6\cos\alpha + 2 = 0$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$8\cos^3\alpha - 5\cos^2\alpha - 6\cos\alpha + 7 = 0$

$(\cos\alpha + 1)(8\cos^2\alpha - 13\cos\alpha + 7) = 0 \Rightarrow \cos\alpha = -1 \Rightarrow \alpha = \pi, -\pi$

$\Delta < 0$

جواب ندارد

۳- $2\sin\alpha\cos 2\alpha + \sin\alpha = 1 \Rightarrow 2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1$

$3\sin\alpha$

$1 - 2\sin^2\alpha$

$\Rightarrow (\sin\alpha + 1)(2\sin\alpha - 1)^2 = 0 \Rightarrow \sin\alpha = -1$ یا $\sin\alpha = \frac{1}{2}$

$\sin\alpha = -1 \Rightarrow \alpha = \frac{3\pi}{2}, \quad \sin\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow \frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{2}$

۴- $\sin(\frac{\pi}{2} + 2x)^{-1} + \cos(\frac{\pi}{2} + 4x)^{-1} \Rightarrow \cos 2x = \sin 4x$

$\Rightarrow \cos 2x = 2\sin 2x\cos 2x \Rightarrow 1 = 2\sin 2\alpha \Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow$

$2\alpha = 2k\pi + \frac{\pi}{6} \Rightarrow \alpha = k\pi + \frac{\pi}{12}$ یا $2\alpha = 2k\pi - \frac{\pi}{6} \Rightarrow \alpha = k\pi + \frac{5\pi}{12}$

$\Rightarrow \alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \Rightarrow \tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$

$\cos(\alpha + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} \Rightarrow \cos^2(\alpha+\frac{\pi}{4}) = \frac{1}{3} \Rightarrow \sin 2\alpha = -\frac{1}{3}$ ‏ـ۵

$\Rightarrow \sin 2\alpha = \frac{1}{3}$ $\quad \frac{1+\cos(2\alpha+\frac{\pi}{2})}{2}$ $\quad m(-\sqrt{2}\sin(\alpha - \frac{\pi}{4}))$

$\Rightarrow (\sqrt{2}\sin(\frac{\pi}{4}-\alpha))m = \frac{\sqrt{6}}{3}$ $\quad m \cdot \sqrt{6} = \sqrt{6}$

$\Rightarrow$ $m = \frac{2\sqrt{6}}{\frac{\sqrt{6}}{3}} \Rightarrow m = 6$

$\sin(\alpha + \frac{\pi}{6})\cos(\alpha - \frac{\pi}{3}) = 1 \Rightarrow \cos(\frac{\pi}{3} - \alpha)\cos(\alpha - \frac{\pi}{3})$ ‏ـ۶

$= 1 \Rightarrow$ $\sin^2(\alpha + \frac{\pi}{6}) = 1 \Rightarrow \sin(\alpha + \frac{\pi}{6}) = \pm 1$

$\Rightarrow \alpha + \frac{\pi}{6} = k\pi + \frac{\pi}{2} \Rightarrow \alpha = k\pi + \frac{\pi}{3}$ ـ ۰ ، در بازه صفر ۲ جواب دارد

$\sin\alpha + \sqrt{3}\cos\alpha = \sqrt{2} \xrightarrow{\div 2} \sin(\alpha + \frac{\pi}{3}) = \frac{\sqrt{2}}{2}$ ‏ـ۷

$\Rightarrow \alpha + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \Rightarrow \alpha = 2k\pi - \frac{\pi}{12}$ یا $\alpha + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4}$

$\Rightarrow \alpha = 2k\pi + \frac{5\pi}{12} \Rightarrow \alpha = \frac{5\pi}{12}, -\frac{\pi}{12}, \frac{23\pi}{12}$

$\frac{5\pi}{12} - \frac{\pi}{12} + \frac{23\pi}{12} = \frac{27\pi}{12}$

$4\sin\alpha\sin\left(\frac{\pi}{2}-\alpha\right)=1 \Rightarrow 4\sin\alpha\cos\alpha=1 \Rightarrow 2\sin 2\alpha=1$

$\Rightarrow \sin 2\alpha=\frac{1}{2} \Rightarrow 2\alpha=2k\pi+\frac{\pi}{6} \Rightarrow \alpha=k\pi+\frac{\pi}{12}$ او $2\alpha=2k\pi+\frac{5\pi}{6}$

$\Rightarrow \alpha=k\pi+\frac{5\pi}{12}$ $\xrightarrow{\text{مجموع}}$ $\frac{\pi}{12}+\frac{13\pi}{12}+\frac{25\pi}{12}+\frac{37\pi}{12}=5\pi$

$\tan 2\alpha\tan\alpha=1 \Rightarrow \tan 2\alpha=\frac{1}{\tan\alpha} \Rightarrow \tan 2\alpha=\tan\left(\frac{\pi}{2}-\alpha\right)$

$\Rightarrow 2\alpha=k\pi+\frac{\pi}{2}-\alpha \xrightarrow{\times 3} 6\alpha=3k\pi+\frac{\pi}{2} \Rightarrow \alpha=\frac{k\pi}{2}+\frac{\pi}{12}$

$\xrightarrow{\text{مجموع}}$ $\frac{\pi}{12}+\frac{7\pi}{12}+\frac{13\pi}{12}+\frac{19\pi}{12}=4\pi$