۱۸,۲۵

$\sin(x+\frac{\pi}{4}) = \pm 1 \rightarrow x+\frac{\pi}{4} = k\pi + \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{4}$

۶

$\sin(x+\frac{\pi}{4})\underbrace{\cos(x-\frac{\pi}{4})}_{\frac{\pi}{4}-x}=1 \Longrightarrow \sin(x+\frac{\pi}{4})\underbrace{\cos(\frac{\pi}{4}-x)}_{\text{متمم زاویه}} = \sin(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=1$ ۲ جواب

متمم زاویه $x+\frac{\pi}{4}$ است پس $\sin$ می‌نویسیم

۱

$\sin\theta = \frac{1}{1} \Rightarrow$

$\Rightarrow \sin^2(x+\frac{\pi}{4})=1 \Rightarrow \sin(x+\frac{\pi}{4}) = \cancel{1}\ \pm 1 \Longrightarrow \begin{cases} x+\frac{\pi}{4} = 2k\pi+\frac{\pi}{4} \Rightarrow x = 2k\pi \\ x+\frac{\pi}{4} = \cancel{2k\pi+\frac{\pi}{4}} \Rightarrow x = 2k\pi + \frac{3\pi}{2} \end{cases}$ ۲ جواب

۷

$\sin x + \sqrt{3}\cos x = \sqrt{3} \longrightarrow \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\theta) \Rightarrow 2\sin(x+\frac{\pi}{3}) = \sqrt{3}$

$\tan\theta = \frac{b}{a} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow$

۳ ✓

$\Rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{3}}{2} \longrightarrow x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \Rightarrow x = 2k\pi - \frac{\pi}{12},\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \Rightarrow x = 2k\pi + \frac{5\pi}{12}$

$x = 2k\pi - \frac{\pi}{12} \xrightarrow{[-\pi,2\pi]\ \text{در}} \{\frac{-\pi}{12}, \frac{23\pi}{12}\},\ x = 2k\pi + \frac{5\pi}{12} \xrightarrow{[-\pi,2\pi]\ \text{در}} \{\frac{5\pi}{12}\} \Longrightarrow \frac{23\pi}{12} + \frac{5\pi}{12} - \frac{\pi}{12} = \frac{27\pi}{12} = \frac{9\pi}{4}$

۸

$4\sin x \underbrace{\sin(\frac{3\pi}{2}-x)}_{-\cos x} = 1 \Longrightarrow -4\underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x} = 1 \Longrightarrow -2\sin 2x = 1 \Longrightarrow \sin 2x = -\frac{1}{2} \Rightarrow$

$\Rightarrow 2x = 2k\pi + \frac{7\pi}{6} \Rightarrow x = k\pi + \frac{7\pi}{12},\ 2x = 2k\pi + \frac{11\pi}{6} \Rightarrow x = k\pi + \frac{11\pi}{12} \Rightarrow$

$x = k\pi + \frac{7\pi}{12} \xrightarrow{[0,2\pi]} x = \{\frac{7\pi}{12}, \frac{19\pi}{12}\},\ x = k\pi + \frac{11\pi}{12} \xrightarrow{[0,2\pi]} x = \{\frac{11\pi}{12}, \frac{23\pi}{12}\}$

$\xrightarrow{\text{جمع}} \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi$

۳ ✓

۹

$\underbrace{(1+\cos 2x)}_{2\cos^2 x}\underbrace{(1+\cos 4x)}_{2\cos^2 2x}\underbrace{(1+\cos 8x)}_{2\cos^2 4x} = \frac{1}{8} \Longrightarrow 8\cos^2 x\cos^2 2x\cos^2 4x = \frac{1}{8}$

$\Longrightarrow \cos^2 x\cos^2 2x\cos^2 4x = \frac{1}{64} \Longrightarrow \cos x\cos 2x\cos 4x = \frac{1}{8} \xrightarrow{\times \sin x} \underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x}\cos 2x\cos 4x = \frac{\sin x}{8}$

$\Longrightarrow \frac{\sin 4x}{8} = \frac{\sin x}{8} \Rightarrow \sin 4x = \sin x \Rightarrow 4x = 2k\pi + x \Rightarrow 3x = 2k\pi \Rightarrow x = \frac{2k\pi}{3}$ $\quad \frac{1}{2}\sin 4x$

$4x = 2k\pi + \pi - x \Rightarrow 5x = 2k\pi + \pi \Rightarrow x = \frac{2k\pi}{5} + \frac{\pi}{5}$ $\quad \frac{1}{2}\cdot\frac{1}{2}\sin 4x$ $\quad \frac{1}{8}\sin 4x$

$\Longrightarrow \max = \frac{8\pi}{9}$

۳ ✓

۱۰

$\tan 2x \tan x = 1 \Longrightarrow \frac{\sin 2x}{\cos 2x} \times \frac{\sin x}{\cos x} = 1 \Longrightarrow \sin 2x \sin x = \cos 2x \cos x$

$\Longrightarrow \sin 2x\sin x - \cos 2x\cos x = 0 \Longrightarrow \cos 4x = 0 \Longrightarrow 4x = 2k\pi \pm \frac{\pi}{2} \begin{cases} x = \frac{k\pi}{2} + \frac{\pi}{8} \\ x = \frac{k\pi}{2} - \frac{\pi}{8} \end{cases}$

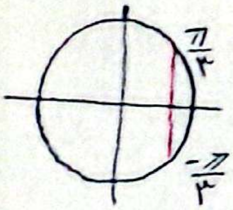
$x \xrightarrow{[\pi,2\pi]\ \text{جواب}} \frac{9\pi}{8} + \frac{11\pi}{8} + \frac{13\pi}{8} + \frac{15\pi}{8} = \frac{48\pi}{8} = 6\pi$

البته جواب (۱) هم صحیح بنظر میرسد

۳ ✓

آرا آزادنیا — تکلیف شماره ۱۷ — دوازدهم تیر A

① $8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = \underbrace{1+\tan^2 x}_{\frac{1}{\cos^2 x}} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$



$\Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

۲

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② $5\underbrace{\sin^2(x)}_{1-\cos^2 x} + 2\underbrace{\cos(3x)}_{2(4\cos^3 x - 3\cos x)} = -2 \Rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x + 2 = 0 \Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

۲

$\Rightarrow 8t^3 - 5t^2 - 6t + 7 = 0 \Rightarrow (t+1)(8t^2 - 13t + 7) = 0 \Rightarrow \cos x = -1 \Rightarrow x = -\pi , \pi$ ۲ جواب

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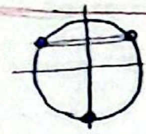
③ $2\sin(x)\underbrace{\cos(2x)}_{1-2\sin^2 x} + \sin(x) = 1 \Rightarrow -4\sin^3 x + 3\sin x - 1 = 0 \Rightarrow -4t^3 + 3t - 1 = 0$



$2k\pi + \frac{5\pi}{6}$

$\Rightarrow \underbrace{(t+1)}_{t=-1}\underbrace{(-4t^2+4t-1)}_{t=\frac{1}{2}} = 0 \Rightarrow \sin x = -1 , \frac{1}{2} \Rightarrow \sin x = -1 \Rightarrow x = 2k\pi + \frac{3\pi}{2} , \sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6}$

۳ جواب دارد.

۱٫۷۵

سوال مجموع جواب رو می‌خواهد.

$\frac{3\pi}{2} + \frac{5\pi}{6} + \frac{\pi}{6} = \frac{5\pi}{2}$

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④ $\frac{1}{\sin(\frac{\pi+4x}{2})} + \frac{1}{\cos(\frac{\pi+8x}{2})} = 0 \Rightarrow \frac{1}{\sin(2x+\frac{\pi}{2})} + \frac{1}{\cos(4x+\frac{\pi}{2})} = 0 \Rightarrow \frac{1}{\cos 2x} + \frac{1}{\sin 4x} = 0$

$\Rightarrow \frac{1}{\cos 2x} + \frac{1}{2\sin 2x\cos 2x} = 0 \Rightarrow \frac{2\sin 2x + 1}{2\sin 2x\cos 2x} = 0 \Rightarrow 2\sin 2x + 1 = 0 \Rightarrow 2\sin 2x = -1 \Rightarrow \sin 2x = \frac{-1}{2}$

$\Rightarrow 2x = 2k\pi + \frac{7\pi}{6} \Rightarrow x = k\pi + \frac{7\pi}{12}$ و $2x = 2k\pi + \frac{11\pi}{6} \Rightarrow x = k\pi + \frac{11\pi}{12}$

$\Rightarrow a = \frac{11\pi}{12} - \frac{7\pi}{12} = \frac{4\pi}{12} \Rightarrow a = \frac{\pi}{3} \Rightarrow \tan 2a = \tan\frac{2\pi}{3} = -\sqrt{3}$

۲

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⑤ $m(\underbrace{\cos x - \sin x}_{\sqrt{2}\cos(\underbrace{x+\frac{\pi}{4}}_{t})}) - 3\sqrt{6}\sin 2x = \sqrt{6} \Rightarrow m\sqrt{2}\underbrace{\cos t}_{\frac{1}{\sqrt{3}}} + 3\sqrt{6}\underbrace{\cos 2t}_{\underbrace{2\cos^2 t}_{\frac{1}{3}} - 1} = \sqrt{6}$

$(\sin(2t - \frac{\pi}{2}))$

$\Rightarrow \frac{m\sqrt{2}}{\sqrt{3}} + (\overbrace{\frac{-3\sqrt{6}}{3}}^{-\sqrt{6}}) = \sqrt{6} \Rightarrow m\sqrt{2} = 2\sqrt{6}\times\sqrt{3} \Rightarrow m = \frac{2\sqrt{6}\times\sqrt{3}}{\sqrt{2}} \Rightarrow \frac{2\overbrace{\sqrt{6}\times\sqrt{3}\times\sqrt{2}}^{\sqrt{36}}}{2} = 6 = m$

۲