(6)

$$\sin\left(x+\frac{\pi}{6}\right)\cos\underbrace{\left(x-\frac{\pi}{3}\right)}_{\frac{\pi}{3}-x}=1 \Longrightarrow \sin\left(x+\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}-x\right)=\sin\left(x+\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)=1$$

متمم زاویه $x+\frac{\pi}{6}$ است پس $\sin$ می‌کنیم

$$\sin\theta=\frac{1}{2}\Rightarrow$$

$$\Rightarrow 2\sin\left(x+\frac{\pi}{6}\right)=1 \Rightarrow \sin\left(x+\frac{\pi}{6}\right)=\frac{1}{2} \Longrightarrow \begin{cases} x+\frac{\pi}{6}=2k\pi+\frac{\pi}{6} \Rightarrow x=2k\pi \\ x+\frac{\pi}{6}=2k\pi+\frac{5\pi}{6} \Rightarrow x=2k\pi+\frac{2\pi}{3} \end{cases}$$ ۲ جواب

---

(7)

$$\sin x+\sqrt{3}\cos x=\sqrt{2} \longrightarrow \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\theta) \Rightarrow 2\sin\left(x+\frac{\pi}{3}\right)=\sqrt{2}$$

$$\tan\theta=\frac{b}{a}=\sqrt{3}\Rightarrow\theta=\frac{\pi}{3}$$

$$\sin\theta=\frac{\sqrt{2}}{2}\Rightarrow$$

$$\Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2} \longrightarrow x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \Rightarrow x=2k\pi-\frac{\pi}{12},\quad x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \Rightarrow x=2k\pi+\frac{5\pi}{12}$$

$$x=2k\pi-\frac{\pi}{12} \xrightarrow{[-\pi,2\pi]\ \text{در}\ x} \left\{\frac{-\pi}{12},\frac{23\pi}{12}\right\},\quad x=2k\pi+\frac{5\pi}{12} \xrightarrow{[-\pi,2\pi]\ \text{در}\ x} \left\{\frac{5\pi}{12}\right\} \Longrightarrow \frac{23\pi}{12}+\frac{5\pi}{12}-\frac{\pi}{12}=\frac{27\pi}{12}=\frac{9\pi}{4}$$

---

(8)

$$4\sin x\sin\underbrace{\left(\frac{3\pi}{2}-x\right)}_{-\cos x}=1 \Longrightarrow -4\underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x}=1 \Rightarrow -2\sin 2x=1 \Rightarrow \sin 2x=-\frac{1}{2} \Rightarrow$$

$$\Rightarrow 2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12},\quad 2x=2k\pi+\frac{11\pi}{6} \Rightarrow x=k\pi+\frac{11\pi}{12} \Rightarrow$$

$$x=k\pi+\frac{7\pi}{12} \xrightarrow{[0,2\pi]} x=\left\{\frac{7\pi}{12},\frac{19\pi}{12}\right\},\quad x=k\pi+\frac{11\pi}{12} \xrightarrow{[0,2\pi]} x=\left\{\frac{11\pi}{12},\frac{23\pi}{12}\right\}$$

$$\xrightarrow{\text{جمع}} \frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=5\pi$$

---

(9)

$$\underbrace{(1+\cos 2x)}_{2\cos^2 x}\underbrace{(1+\cos 4x)}_{2\cos^2 2x}\underbrace{(1+\cos 8x)}_{2\cos^2 4x}=\frac{1}{8} \Rightarrow 8\cos^2 x\cos^2 2x\cos^2 4x=\frac{1}{8}$$

$$\Rightarrow \cos^2 x\cos^2 2x\cos^2 4x=\frac{1}{64} \Rightarrow \cos x\cos 2x\cos 4x=\frac{1}{8} \xrightarrow{\times\sin x} \underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x}\cos 2x\cos 4x=\frac{\sin x}{8}$$

$$\frac{1}{2}\cdot\frac{1}{2}\sin 4x \qquad \frac{1}{8}\sin 4x$$

$$\Rightarrow \frac{\sin 4x}{8}=\frac{\sin x}{8} \Rightarrow \sin 4x=\sin x \Rightarrow \begin{cases} 4x=2k\pi+x \Rightarrow 3x=2k\pi \Rightarrow x=\frac{2k\pi}{3} \\ 4x=2k\pi+\pi-x \Rightarrow 5x=2k\pi+\pi \Rightarrow x=\frac{2k\pi}{5}+\frac{\pi}{5} \end{cases}$$

$$\Rightarrow \max=\frac{8\pi}{9}$$

---

(10)

$$\tan 3x\tan x=1 \Longrightarrow \frac{\sin 3x}{\cos 3x}\times\frac{\sin x}{\cos x}=1 \Longrightarrow \sin 3x\sin x=\cos 3x\cos x$$

$$\Rightarrow \sin 3x\sin x-\cos 3x\cos x=0 \Longrightarrow \cos 4x=0 \Longrightarrow 4x=2k\pi\pm\frac{\pi}{2} \begin{cases} x=\frac{k\pi}{2}+\frac{\pi}{8} \\ x=\frac{k\pi}{2}-\frac{\pi}{8} \end{cases}$$

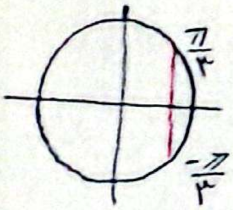
$$x \xrightarrow{[\pi,2\pi]\ \text{بازه}} \frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=6\pi$$

البته جواب (گزینه صحیح) را ندارد.

دوازدهم تجربی A — تکلیف شماره ۱۷ — آرا آزادنیا

(۱)

$$8\cos x-\tan^2 x=1 \Rightarrow 8\cos x=\underbrace{1+\tan^2 x}_{\frac{1}{\cos^2 x}} \Rightarrow 8\cos^3 x=1 \Rightarrow \cos^3 x=\frac{1}{8} \Rightarrow \cos x=\frac{1}{2}$$



$$\Rightarrow x=2k\pi \pm \frac{\pi}{3}$$

---

(۲)

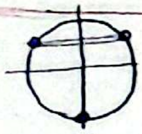
$$5\underbrace{\sin^2(x)}_{1-\cos^2 x}+2\underbrace{\cos(3x)}_{2(4\cos^3 x-3\cos x)}=-2 \Longrightarrow 5-5\cos^2 x+8\cos^3 x-6\cos x+2=0 \Rightarrow 8\cos^3 x-5\cos^2 x-6\cos x+7=0$$

$$\Rightarrow 8t^3-5t^2-6t+7=0 \Rightarrow (t+1)(8t^2-13t+7)=0 \Rightarrow \cos x=-1 \Rightarrow x=-\pi \text{ و } \pi$$ ۲ جواب

---

(۳)

$$2\sin(x)\underbrace{\cos(2x)}_{1-2\sin^2 x}+\sin(x)=1 \Rightarrow -4\sin^3 x+3\sin x-1=0 \Rightarrow -4t^3+3t-1=0$$



$$\Rightarrow \underbrace{(t+1)}_{t=-1}\underbrace{(-4t^2+4t-1)}_{t=\frac{1}{2}}=0 \Rightarrow \sin x=-1 \text{ و } \frac{1}{2} \Rightarrow \sin x=-1 \Rightarrow x=2K\pi+\frac{3\pi}{2},\ \sin x=\frac{1}{2} \Rightarrow x=2K\pi+\frac{\pi}{6}$$

۳ جواب دارد.

---

(۴)

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)}+\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}=0 \Rightarrow \frac{1}{\sin(2x+\frac{\pi}{2})}+\frac{1}{\cos(4x+\frac{\pi}{2})}=0 \Rightarrow \frac{1}{\cos 2x}+\frac{1}{\sin 4x}=0$$

$$\Rightarrow \frac{1}{\cos 2x}+\frac{1}{2\sin 2x\cos 2x}=0 \Rightarrow \frac{2\sin 2x+1}{2\sin 2x\cos 2x}=0 \Rightarrow 2\sin 2x+1=0 \Rightarrow 2\sin 2x=-1 \Rightarrow \sin 2x=\frac{-1}{2}$$

$$\Rightarrow 2x=2K\pi+\frac{7\pi}{6} \Rightarrow x=K\pi+\frac{7\pi}{12} \text{ و } 2x=2K\pi+\frac{11\pi}{6} \Rightarrow x=K\pi+\frac{11\pi}{12}$$

$$\Rightarrow a=\frac{11\pi}{12}-\frac{7\pi}{12}=\frac{4\pi}{12} \Rightarrow a=\frac{\pi}{3} \Rightarrow \tan 2a=\tan\frac{2\pi}{3}=-\sqrt{3}$$

---

(۵)

$$m\underbrace{(\cos x-\sin x)}_{\underbrace{\sqrt{2}\cos(x+\frac{\pi}{4})}_{t}}-3\sqrt{6}\underbrace{\sin 2x}_{(\sin(2t-\frac{\pi}{2}))}=\sqrt{6} \Rightarrow m\sqrt{2}\underbrace{\cos t}_{\frac{1}{\sqrt{3}}}+3\sqrt{6}\underbrace{\cos 2t}_{\underbrace{2\cos^2 t-1}_{\frac{1}{3}}}=\sqrt{6}$$

$$\Rightarrow \frac{m\sqrt{2}}{\sqrt{3}}+\overbrace{\left(\frac{-3\sqrt{6}}{3}\right)}^{-\sqrt{6}}=\sqrt{6} \Rightarrow m\sqrt{2}=\cancel{\sqrt{3}}\ 2\sqrt{6}\times\sqrt{3} \Rightarrow m=\frac{2\sqrt{6}\times\sqrt{3}}{\sqrt{2}} \Rightarrow \frac{2\overbrace{\sqrt{6}\times\sqrt{3}\times\sqrt{2}}^{\sqrt{36}}}{2}=6=m$$