Subject:　　　　　　Date:

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مثلثات بهروز

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(در واقع یک سوال rank درسی کلاس)

$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1 + \tan^2 x = \frac{1}{\cos^2 x} \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow x = 2k\pi \pm \frac{\pi}{3}$ ~ $\boxed{\text{۳ جواب}}$

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$\Rightarrow 5\sin^2 x = -2\cos^3 x - 2$

$0 \leq \sin^2 x \leq 1 \Rightarrow 0 \leq 5\sin^2 x \leq 5 \Rightarrow 0 \leq -2\cos^3 x - 2 \leq 5 \Rightarrow 2 \leq -2\cos^3 x \leq 7$ (در صورتی) میدانیم

$\Rightarrow -1 \geq \cos^3 x \geq -\frac{7}{2}$

$\forall x; x \in \mathbb{R}: \cos^3 x \geq -1$

$\Rightarrow \cos^3 x = -1 \Rightarrow 3x = 2k\pi + \pi \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

k	...	-2	-1	0	1	...
x	×	$-\pi$	$-\frac{\pi}{3}$	$\frac{\pi}{3}$	π	×
		✓	×	×	✓	

$\boxed{\text{۲ جواب}}$

جواب نمی‌دهد در معادله

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$2\sin x \cos 2x + \sin x = 1 \Rightarrow \sin x(2\underbrace{\cos 2x}_{1-2\sin^2 x} + 1) = \sin x(3 - 4\sin^2 x)$

$\Rightarrow 3\sin x - 4\sin^3 x = 1 \Rightarrow \sin 3x = 1 \Rightarrow 3x = 2k\pi + \frac{\pi}{2} \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$

k	...	0	1	2	...
x	×	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{3\pi}{2}$	×

$\boxed{\text{۳ جواب}}$

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$\Rightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0$　　$-\sin 4x = \cos(\frac{\pi}{2} + 4x)$, $\sin(\frac{\pi}{2} + 2x) = \cos 2x$

$\Rightarrow \frac{1}{\cos 2x} - \frac{1}{2\sin 2x \cos 2x} = 0 \Rightarrow \frac{1}{\cos 2x}\left(1 - \frac{1}{2\sin 2x}\right) = \frac{1}{\cos 2x}\left(\frac{2\sin 2x - 1}{2\sin 2x}\right) = 0$

$\Rightarrow 2\sin 2x - 1 = 0 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow \begin{cases} 2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = k\pi + \frac{\pi}{12} \xrightarrow{x \in [0,\pi]} x = \frac{\pi}{12} \\ 2x = 2k\pi + \pi - \frac{\pi}{6} \Rightarrow x = k\pi + \frac{5\pi}{12} \xrightarrow{x \in [0,\pi]} x = \frac{5\pi}{12} \end{cases}$

$\frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \boxed{\frac{\pi}{3}}$

PINE NOTEBOOK

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$$\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}} \Rightarrow \cos x\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}-\sin x\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}=\frac{1}{\sqrt{2}}(\cos x-\sin x)=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \cos x-\sin x=\sqrt{\frac{2}{3}} \ ① \xrightarrow{\text{توان ۲}} (\cos x-\sin x)^2=\left(\sqrt{\frac{2}{3}}\right)^2 \Rightarrow 1-\sin 2x=\frac{2}{3} \Rightarrow \sin 2x=\frac{1}{3} \ ②$$

$$m(\cos x-\sin x)-3\sqrt{6}\sin(2x)=\sqrt{6} \underset{②,①}{\Rightarrow} m\left(\frac{\sqrt{2}}{\sqrt{3}}\right)-\frac{3\sqrt{6}}{3}=\sqrt{6} \Rightarrow \frac{\sqrt{6}m}{3}-\sqrt{6}=\sqrt{6}$$

$$\Rightarrow \frac{\sqrt{6}m}{3}=2\sqrt{6} \Rightarrow \boxed{m=6}$$

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$$\cos\left(x-\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}-x\right)=\sin\left(\frac{\pi}{2}-\left(\frac{\pi}{3}-x\right)\right)=\sin\left(x+\frac{\pi}{6}\right)$$

$$\Rightarrow \sin^2\left(x+\frac{\pi}{6}\right)=1 \Rightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1 \Rightarrow x+\frac{\pi}{6}=k\pi+\frac{\pi}{2} \Rightarrow x=k\pi+\frac{\pi}{3} \xrightarrow{x\in[0,2\pi]} x=\frac{\pi}{3},\ x=\frac{4\pi}{3}$$

$\boxed{\text{۲ جواب}}$

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$$\Rightarrow \underbrace{\frac{1}{2}}_{\cos\frac{\pi}{3}}\sin x+\underbrace{\frac{\sqrt{3}}{2}}_{\sin\frac{\pi}{3}}\cos x=\frac{\sqrt{2}}{2} \Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}\right) \Rightarrow \begin{cases} x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \\ x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4} \end{cases}$$

$$\Rightarrow \begin{cases} x=2k\pi-\frac{\pi}{12} \\ x=2k\pi+\frac{5\pi}{12} \end{cases}$$

k	...	0	1	...
x	×	$-\frac{\pi}{12}$ ✓	$\frac{23\pi}{12}$ ✓	×

k	...	−1	0	1	...
x	...	$-\frac{19\pi}{12}$ ×	$\frac{5\pi}{12}$ ✓	$\frac{29\pi}{12}$ ×	×

$\left(-\frac{19\pi}{12}\right)$

$$\text{جمع جوابها}=-\frac{\pi}{12}+\frac{5\pi}{12}+\frac{23\pi}{12}=\frac{27\pi}{12}=\frac{9\pi}{4}$$

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$$4\sin x\underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x}=1 \Rightarrow -2(\underbrace{2\sin x\cos x}_{\sin 2x})=1 \Rightarrow \sin 2x=-\frac{1}{2}$$

$$\Rightarrow \begin{cases} 2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12} \\ 2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12} \end{cases}$$

k	...	1	2	...
x	×	$\frac{11\pi}{12}$	$\frac{23\pi}{12}$	×

k	...	0	1	...
x	×	$\frac{7\pi}{12}$	$\frac{19\pi}{12}$	×

$$\text{جمع جوابها}=\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$$

9) فرمول طلایی: $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

$$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{2\cos^2 4\alpha} = \frac{1}{8}$$

$\Rightarrow 64\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = 1 \Rightarrow 8\cos\alpha\cos 2\alpha\cos 4\alpha = \pm 1 \xRightarrow{\times \sin\alpha} 8\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha = \pm\sin\alpha$

$\Rightarrow 4\sin 2\alpha\cos 2\alpha\cos 4\alpha = \pm\sin\alpha \Rightarrow 2\sin 4\alpha\cos 4\alpha = \pm\sin\alpha \Rightarrow \sin 8\alpha = \pm\sin\alpha$

$$\Rightarrow \begin{cases} \sin 8\alpha = \sin\alpha \Rightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \Rightarrow 7\alpha = 2k\pi \Rightarrow \alpha = \frac{2k\pi}{7} \\ 8\alpha = 2k\pi + \pi - \alpha \Rightarrow 9\alpha = (2k+1)\pi \Rightarrow \alpha = \frac{(2k+1)\pi}{9} \end{cases} \\ \sin 8\alpha = -\sin\alpha = \sin(-\alpha) \Rightarrow \begin{cases} 8\alpha = 2k\pi - \alpha \Rightarrow 9\alpha = 2k\pi \Rightarrow \alpha = \frac{2k\pi}{9} \\ 8\alpha = 2k\pi + \pi + \alpha \Rightarrow 7\alpha = (2k+1)\pi \Rightarrow \alpha = \frac{(2k+1)\pi}{7} \end{cases} \end{cases}$$

$\alpha = \frac{k\pi}{7}$ ، $\alpha = \frac{k\pi}{9}$

$\Rightarrow$

$\alpha = \frac{k\pi}{7}$

k	0	1	2	3	4	5	6	7
x	1	$\frac{\pi}{7}$	$\frac{2\pi}{7}$	$\frac{3\pi}{7}$	$\frac{4\pi}{7}$	$\frac{5\pi}{7}$	$\frac{6\pi}{7}$	π

$\alpha = \frac{k\pi}{9}$

k	0	1	2	3	4	5	6	7	8	9
x	0	$\frac{\pi}{9}$	$\frac{2\pi}{9}$	$\frac{\pi}{3}$	$\frac{4\pi}{9}$	$\frac{5\pi}{9}$	$\frac{2\pi}{3}$	$\frac{7\pi}{9}$	$\frac{8\pi}{9}$	π

لذا طبق این اعداد، π بیشترین است (اما در رابطه صدق نمی‌کند) ($8=\frac{1}{8}$) لذا ماکزیمم عضو A $\frac{8\pi}{9}$ (که در رابطه صدق کند)

10) $\tan 3x \tan x = 1 \Rightarrow \tan 3x = \frac{1}{\tan x} = \cot x = \tan(\frac{\pi}{2} - x)$

$\Rightarrow 3x = k\pi + \frac{\pi}{2} - x \Rightarrow 4x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$

k	...	3	4	5	6	7	8
x	x	$\frac{7\pi}{8}$ ✗	$\frac{9\pi}{8}$ ✓	$\frac{11\pi}{8}$ ✓	$\frac{13\pi}{8}$ ✓	$\frac{15\pi}{8}$ ✓	$\frac{17\pi}{8}$ ✗

(با توجه به بازه صورت سوال حل شد)

$S = \frac{9\pi}{8} + \frac{11\pi}{8} + \frac{13\pi}{8} + \frac{15\pi}{8} = \frac{48\pi}{8} = \boxed{6\pi}$

(مجموع جواب‌ها)