نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$8\cos x - 4\tan^2 x = 1 \rightsquigarrow 8\cos x = 4\tan^2 x + 1 = \frac{4}{\cos^2 x} + 1$

$\rightarrow 8\cos^3 x = 1 \rightarrow 2\cos x = 1 \rightsquigarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3} \rightsquigarrow x = 2k\pi \pm \frac{\pi}{3}$

$D_f = [0, 2\pi]$ $\quad \frac{\pi}{3}$ $\quad 2\pi - \frac{\pi}{3}$

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$5\sin^2 x + 2\cos(2x) = -2 \rightsquigarrow 5\sin^2 x + 2(\cos 2x + 1) = 0$

$5\sin^2 x + 4\cos^2 x = 0 \xrightarrow{\sin^2 x, \cos^2 x > 0} 5\sin^2 x = 0 \rightarrow \sin x = 0$

$x = k\pi \rightarrow (-\pi), 0, (\pi)$

$4\cos^2\left(\frac{3x}{2}\right) = 0 \rightsquigarrow \cos\frac{3x}{2} = 0$

$\frac{3x}{2} = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{4k\pi}{3} + \frac{\pi}{3}$

$x = \frac{4k\pi}{3} + \frac{\pi}{3} \rightarrow (-\pi), -\frac{\pi}{3}, \frac{\pi}{3}$

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$2\sin x \cos 2x + \sin x = 1 \rightarrow \sin x(2\cos 2x + 1) = 1$

$\sin x(2\cos^2 x - 1 + 2\cos^2 x) = 1 \rightarrow \sin x(4\cos^2 x - 1) = 1$

$\cos^2 x = 1 - \sin^2 x \rightarrow \sin x(4 - 4\sin^2 x - 1) = 1 \rightarrow -4\sin^3 x + 3\sin x = 1$

$\rightarrow \sin(3x) = 1 \rightsquigarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$ $\quad \frac{\pi}{6}$ $\quad \frac{5\pi}{6}$ $\rightarrow$ دو جواب

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$\frac{1}{\sin\left(\frac{\pi}{2} + 2x\right)} = -\frac{1}{\cos\left(\frac{\pi}{2} + 4x\right)} \rightarrow \sin\left(\frac{\pi}{2} + 2x\right) = -\cos\left(\frac{\pi}{2} + 4x\right)$

$\cos 2x = -\sin 4x$

$\cos x \neq 0 \rightarrow 2x \neq k\pi + \frac{\pi}{2} \rightarrow x \neq \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 2x = \frac{1}{2} = \sin\frac{\pi}{6}$

$2x = 2k\pi + \frac{\pi}{6} \rightsquigarrow x = k\pi + \frac{\pi}{12}$ $\quad \frac{\pi}{12}$

$2x = 2k\pi + \pi - \frac{\pi}{6} \rightsquigarrow x = k\pi + \frac{5\pi}{12} \rightarrow \frac{5\pi}{12}$

$\rightarrow \alpha = \frac{5\pi}{12} = \frac{\pi}{4} \rightarrow \tan\frac{\pi}{4} = 1$

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$\sin(x+\frac{\pi}{6})$

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$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1 \rightarrow \sin^2(x+\frac{\pi}{6}) = 1$ ($\frac{\pi}{3}-x$)

$\rightarrow \sin(x+\frac{\pi}{6}) = 1 \rightarrow x+\frac{\pi}{6} = 2k\pi+\frac{\pi}{2} \rightarrow x = 2k\pi+\frac{\pi}{3}$ ①

$\sin(x+\frac{\pi}{6}) = -1 \rightarrow x+\frac{\pi}{6} = 2k\pi-\frac{\pi}{2} \rightarrow x = 2k\pi-\frac{2\pi}{3}$ ②

$\frac{\pi}{3}$, $2\pi-\frac{2\pi}{3}$ → ۲ جواب

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$\sin x + \sqrt{3}\cos x = \sqrt{2}$ $\qquad \frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)$, $\tan\alpha = \frac{b}{a}$

$\sqrt{1+3}$ $\quad 2\sin(x+\alpha) \rightarrow 2\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \rightarrow$

$\tan\alpha = \sqrt{3} \rightarrow \alpha = \frac{\pi}{3}$

$\rightarrow x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \rightarrow x = 2k\pi+\frac{\pi}{4}-\frac{\pi}{3} \rightarrow \frac{\pi}{4}-\frac{\pi}{3}$, $2\pi+\frac{\pi}{4}-\frac{\pi}{3}$

$\rightarrow x+\frac{\pi}{3} = 2k\pi+\pi-\frac{\pi}{4} \rightarrow x = 2k\pi+\pi-\frac{\pi}{4}-\frac{\pi}{3}$ $\quad \pi-\frac{\pi}{4}-\frac{\pi}{3}$ → مجموع $\frac{9\pi}{4}$

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$4\sin x \sin(\frac{3\pi}{2}-x) = 1 \rightarrow -4\sin x\cos x = 1 \rightarrow -2\sin 2x = 1$ ($-\cos x$)

$\rightarrow \sin 2x = -\frac{1}{2} = \sin(-\frac{\pi}{6}) \rightarrow 2x = 2k\pi-\frac{\pi}{6} \rightarrow x = k\pi-\frac{\pi}{12}$ ①

$2x = 2k\pi+\pi+\frac{\pi}{6} = \frac{7\pi}{6} \rightarrow x = k\pi+\frac{7\pi}{12}$ ②

① $\pi-\frac{\pi}{12}$, $2\pi-\frac{\pi}{12}$

② $\pi+\frac{7\pi}{12}$, $\frac{7\pi}{12}$ → مجموع 5π

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$(1+\cos(2\alpha))(1+\cos(4\alpha))(1+\cos(8\alpha)) = \frac{1}{8}$

$2\cos^2\alpha \quad 2\cos^2 2\alpha \quad 2\cos^2 4\alpha \quad \frac{1}{4}\sin^2 2\alpha$

$\rightarrow (\cos^2\alpha)(\cos^2 2\alpha)(\cos^2 4\alpha) = \frac{1}{64} \rightarrow \frac{(\sin^2\alpha\cos^2\alpha)(\cos^2 2\alpha)(\cos^2 4\alpha)}{\sin^2\alpha} = \frac{1}{64}$

$\frac{(\sin^2 2\alpha\cos^2 2\alpha)\cos^2 4\alpha}{\sin^2\alpha} = \frac{1}{16} \rightarrow \frac{\frac{1}{4}\sin^2 8\alpha}{\sin^2\alpha} = \frac{1}{4} = 1 \rightarrow \sin^2 8\alpha = \sin^2\alpha$

[redacted] $\rightarrow \frac{1-\cos 16\alpha}{2} = \frac{1-\cos 2\alpha}{2} \rightarrow \cos 16\alpha = \cos 2\alpha$

$\alpha = \frac{k\pi}{7}$

$\alpha = \frac{k\pi}{9} \rightarrow \frac{8\pi}{9}$ = بزرگترین

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$\tan(3x)\tan(x) = 1 \rightarrow \tan 3x = \frac{1}{\tan x} = \cot x = \tan(\frac{\pi}{2}-x)$

$3x = k\pi+\frac{\pi}{2}-x \rightarrow 4x = k\pi+\frac{\pi}{2} \rightarrow x = \frac{2k\pi}{8}+\frac{\pi}{8}$

$\rightarrow \frac{9\pi}{8}$, $\frac{11\pi}{8}$, $\frac{13\pi}{8}$, $\frac{15\pi}{8}$ مجموع $\frac{48\pi}{8} = 6\pi$