نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس ....

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$8\cos x = 1+\tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \xrightarrow{\cos^2 x \neq 0} 8\cos^3 x = 1 \Rightarrow$

$\cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow x = 2k\pi + \frac{\pi}{3} \to \frac{\pi}{3}$

$\Rightarrow x = 2k\pi - \frac{\pi}{3} \to \frac{5\pi}{3}$ } ۲ جواب

(۲) ✓

۲

$\cos^2 \frac{3x}{2} = \frac{1+\cos 3x}{2}$

$5\sin^2 x + 2\left(2\cos^2 \frac{3x}{2} - 1\right) = -2$

$5\sin^2 x + 4\cos^2 \frac{3x}{2} = 0$

دو عبارت نامنفی صفر پس هر عبارت صفر است.

$x = \pi, 0, 5\pi$

$x = 0, -\pi, \pi, -\frac{\pi}{3}$

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$2\sin x(1-2\sin^2 x) + \sin x = 1 \Rightarrow 2\sin x - 4\sin^3 x + \sin x = 1$

$\frac{5\pi}{2} = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2}$

سوال مجموع جواب ها را می‌خواهد.

$\Rightarrow 3\sin x - 4\sin^3 x = 1 \Rightarrow \sin 3x = 1 = \sin\frac{\pi}{2}$ ۱۷۵

$\to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6}$

$\to 3x = 2k\pi + \pi - \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6}$ } $\to \left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ ۳ جواب

۴

$\frac{1}{\sin\left(\frac{\pi}{2}+4x\right)} = \frac{1}{\cos\left(\frac{\pi}{2}+2x\right)} \xrightarrow{\sin 4x \neq 0} -\sin 4x = -\sin\left(\frac{\pi}{2}+2x\right)$

$\sin 4\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow 4x = 2k\pi + \frac{\pi}{2} + 2x \to 2x = 2k\pi + \frac{\pi}{2} \to x = k\pi + \frac{\pi}{4} \to \frac{\pi}{4}$ غ ق ق

$\Rightarrow 4x = 2k\pi + \pi - \frac{\pi}{2} - 2x \to 6x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{6} + \frac{\pi}{12} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

$\tan\frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$ (۲) ✓ $\frac{\pi}{3}$ اضافات

۵

$m(\cos x - \sin x) = \sqrt{6}(1+3\sin 2x) \to -\sqrt{2}\, m \sin\left(x - \frac{\pi}{4}\right) = \sqrt{6}(1+3\sin 2x)$

$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3}$

$\frac{\sqrt{2}}{2} = \cos\left(x+\frac{\pi}{4}\right)$

$-\frac{\sqrt{2}m}{3} = \sqrt{6}(1+3\sin 2x)$

(۱)

$\sin\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{3}}{3} \Rightarrow \sin x \cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x = \frac{\sqrt{3}}{3}$

$\frac{\sqrt{2}}{2}(\sin x - \cos x) = \frac{\sqrt{3}}{3}$

$(\cos x - \sin x)^2 = \frac{\sqrt{4}}{3}$

$\sin 2x = \frac{1}{3}$

$m \times \frac{\sqrt{4}}{3} - 3\sqrt{4} \times \frac{1}{3} = \sqrt{4}$

$m = 4$

۶

$$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)=1$$

$$\rightarrow \sin\left(\frac{\pi}{2}+x-\frac{\pi}{3}\right)\cdot\cos\left(x-\frac{\pi}{3}\right)=\sin\left(x+\frac{\pi}{6}\right)$$

$$\Rightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1$$

$$\sin\left(x+\frac{\pi}{6}\right)=+1 \rightarrow x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2} \rightarrow x=2k\pi+\frac{\pi}{3}$$

$$\sin\left(x+\frac{\pi}{6}\right)=-1 \rightarrow x+\frac{\pi}{6}=2k\pi-\frac{\pi}{2} \rightarrow x=2k\pi-\frac{2\pi}{3}$$

۷

$$a\sin x+b\cos x=\frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)$$

$$\tan\alpha=\frac{b}{a}=\frac{\sqrt{3}}{1}=\sqrt{3} \rightarrow \alpha=\frac{\pi}{3}$$

$$\sin x+\sqrt{3}\cos x=2\sin\left(x+\frac{\pi}{3}\right)$$

$$\Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{1}{2}=\sin\frac{\pi}{6}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{6} \rightarrow x=2k\pi-\frac{\pi}{6}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{5\pi}{6} \rightarrow x=2k\pi+\frac{\pi}{2}$$

$$\sin\left(x+\frac{\pi}{2}\right)=\frac{\sqrt{2}}{2}$$

$$x+\frac{\pi}{2}=2k\pi+\frac{3\pi}{4} \rightarrow x=2k\pi+\frac{\pi}{4}$$

$$x+\frac{\pi}{2}=2k\pi+\frac{5\pi}{4} \rightarrow x=2k\pi+\frac{3\pi}{4}$$

۸

$$4\sin x\sin\left(\frac{\pi}{2}-x\right)=1 \Rightarrow -4\sin x\cos x=1$$

$$-2\sin 2x=1 \rightarrow \sin 2x=-\frac{1}{2}=\sin\frac{7\pi}{6}$$

$$2x=2k\pi+\frac{7\pi}{6} \rightarrow x=k\pi+\frac{7\pi}{12}$$

$$2x=2k\pi-\frac{\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12}$$

$$\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{19\pi}{12}+\frac{35\pi}{12}=5\pi$$

۹

$$(1+\cos 2x)(1+\cos 4x)(1+\cos 8x)=\frac{1}{8}$$

$$\sin^2 x\,(2\cos^2 x)(2\cos^2 2x)(2\cos^2 4x)=\frac{1}{8}$$

$$\sin^2 x\,(\sin^2 2x)(\cos^2 4x)=\frac{1}{8}$$

$$\sin^2 x\cdot 4\sin^2 x\cos^2 x\cdot\cos^2 4x=\frac{1}{8}$$

$$4\sin^4 x\cos^2 x\cos^2 4x=\frac{1}{8}$$

$$\sin^4 x\cos^2 x\cos^2 4x=\frac{1}{32}$$

$$\sin^2 x\cos^2 x\cdot\sin^2 x\cos^2 4x=\frac{1}{32}$$

$$\frac{1}{4}\sin^2 2x\cdot\sin^2 x\cos^2 4x=\frac{1}{32}$$

$$\sin^2 2x\cdot\sin^2 x\cos^2 4x=\frac{1}{8}$$

$$\sin^2 8x=\frac{1}{8}$$

۱۰

$$\tan 2\alpha=\frac{1}{\tan\alpha}=\cot\alpha=\tan\left(\frac{\pi}{2}-\alpha\right)$$

$$2\alpha=k\pi+\frac{\pi}{2}-\alpha$$

$$\alpha=\frac{k\pi}{3}+\frac{\pi}{6}$$

k	0	1	2
α	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{9\pi}{6}$

$$\frac{\pi}{6}+\frac{5\pi}{6}+\frac{9\pi}{6}=3\pi$$

$\sin 2\alpha = \pm \sin \alpha$

$2\alpha = 2k\pi + \alpha$, $2\alpha = 2k\pi + \pi - \alpha$

$2\alpha = 2k\pi - \alpha$ $2\alpha = 2k\pi + \pi + \alpha$

$\max A = \frac{2\pi}{2}$