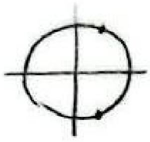
نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ..۱۷.. کلاس ......[illegible] ۸......

۱

$8\cos\alpha = 1+\tan^2\alpha = \frac{1}{\cos^2\alpha}$

$8 = \frac{1}{\cos^3\alpha}\quad (\cos\alpha \neq 0) \longrightarrow \cos\alpha = \frac{1}{2}$



۲ جواب

۲

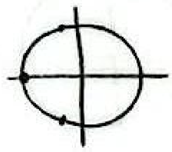
$5\sin^2x + 2\cos 3x = -2 \to 5(1-\cos^2x) + 2(4\cos^3x - 3\cos x) = -2$

$\to 8\cos^3x - 5\cos^2x - 6\cos x + 7 = 0 \to (\cos x+1)(8\cos^2x - 13\cos x - 7) = 0$

$\cos x = -1$

$\cos x = \frac{13+\sqrt{393}}{16} = \frac{32.5}{16} \simeq 2$

$\cos x = \frac{13-\sqrt{393}}{16} = \frac{-6}{16} = -\frac{3}{8}$



۳ جواب

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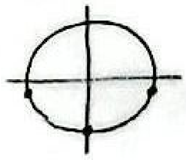
$\sin x(2\cos 2x+1) = 1 \longrightarrow \sin x(2-4\sin^2x+1) = 1 \to 3\sin x - 4\sin^3x = 1$

$\longrightarrow 4\sin^3x - 3\sin x + 1 = 0 \longrightarrow \begin{cases} \sin x = -1 \\ \sin x = \frac{1+\sqrt2}{2} \simeq 1.25 \\ \sin x = \frac{1-\sqrt2}{2} \simeq -0.25 \end{cases}$

$(t+1)(4t^2-4t-1) = 0$

$t = -1 \qquad t = \frac{1\pm\sqrt2}{2}$



۳ جواب

۴

$\frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0 \longrightarrow \frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} = 0 \longrightarrow \sin 4x = \cos 2x = \sin(\frac{\pi}{2} - 2x)$

$4x = 2k\pi + \frac{\pi}{2} - 2x \to x = \frac{k\pi}{3} + \frac{\pi}{12} \to \frac{\pi}{12}, \frac{5\pi}{12}$

$\alpha = \frac{\pi}{6}$

$4x = 2k\pi + \pi - \frac{\pi}{2} + 2x \to x = k\pi + \frac{\pi}{4} \to \frac{\pi}{4}$

$\tan 2\alpha = \tan\frac{\pi}{3} = \boxed{\sqrt3}$

۵

$\cos(x+\frac{\pi}{4}) = \frac{\sqrt2}{2}\cos x - \frac{\sqrt2}{2}\sin x = \frac{\sqrt3}{3} \to \cos x - \sin x = \frac{\sqrt6}{3}$

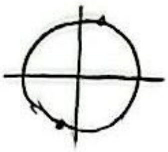
$\frac{\sqrt6}{3}m - 3\sqrt6\sin 2x = \sqrt6 \to m - 9\sin 2x = 1 \to \frac{m-1}{9} = \sin 2x = \frac{1}{3} \to \boxed{m=4}$

$1 - \sin 2x = \frac{2}{3} \to \sin 2x = \frac{1}{3}$

۶

$$\begin{cases}\sin(x+\frac{\pi}{6})\cos(\frac{\pi}{3}-x)=1\\ \sin(x+\frac{\pi}{6})\sin(x+\frac{\pi}{6})=1\end{cases} \longrightarrow \begin{cases}\sin(x+\frac{\pi}{6})=1 \to x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2}\\ \sin(x+\frac{\pi}{6})=-1 \to x+\frac{\pi}{6}=2k\pi+\frac{3\pi}{2}\end{cases}$$

$$x=2k\pi+\frac{\pi}{3}$$
$$x=2k\pi+\frac{4\pi}{3}$$



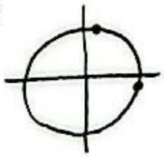
۲ جواب

۷

$$\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2} \longrightarrow \sin(x+\frac{\pi}{3})=\sin\frac{\pi}{4}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \to x=2k\pi-\frac{\pi}{12} \to -\frac{\pi}{12} \quad \frac{23\pi}{12}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \to x=2k\pi+\frac{5\pi}{12} \to \frac{5\pi}{12}$$



$$-\frac{\pi}{12}+\frac{5\pi}{12}+\frac{23\pi}{12}=\frac{27\pi}{12}=\boxed{\frac{9\pi}{4}}$$

۸

$$-4\sin x\cos x=1 \longrightarrow \sin 2x=-\frac{1}{2}$$

$$2x=2k\pi+\frac{7\pi}{6} \to x=k\pi+\frac{7\pi}{12} \to \frac{7\pi}{12} \quad \frac{19\pi}{12}$$

$$2x=2k\pi+\frac{11\pi}{6} \to x=k\pi+\frac{11\pi}{12} \to \frac{11\pi}{12} \quad \frac{23\pi}{12}$$

$$\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$$

۹

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \longrightarrow 2\cos^2\alpha \times 2\cos^2 2\alpha \times 2\cos^2 4\alpha=\frac{1}{8}$$

$$\frac{2\sin^2\alpha}{2\sin^2\alpha}\times 2\cos^2\alpha\times 2\cos^2 2\alpha\times 2\cos^2 4\alpha=\frac{1}{8} \qquad \frac{\sin^2 8\alpha}{\sin^2\alpha}=\frac{1}{8} \longrightarrow \sin^2 8\alpha=\sin^2\alpha$$

$$\longrightarrow \begin{cases}\sin 8\alpha=\sin\alpha \to & 8\alpha=2k\pi+\alpha \longrightarrow \alpha=\frac{2k\pi}{7} \longrightarrow 0 \quad \frac{2\pi}{7} \quad \frac{4\pi}{7} \quad \frac{6\pi}{7}\\ & 8\alpha=2k\pi+\pi-\alpha \longrightarrow \alpha=\frac{2k\pi}{9}+\frac{\pi}{9} \to \frac{\pi}{9} \quad \frac{3\pi}{9} \quad \frac{5\pi}{9} \quad \frac{7\pi}{9} \quad \pi\\ & 8\alpha=2k\pi-\alpha \longrightarrow \alpha=\frac{2k\pi}{9} \to 0 \quad \frac{2\pi}{9} \quad \frac{4\pi}{9} \quad \frac{6\pi}{9} \quad \frac{8\pi}{9}\\ \sin 8\alpha=\sin(-\alpha) \to & 8\alpha=2k\pi+\pi+\alpha \longrightarrow \alpha=\frac{2k\pi}{7}+\frac{\pi}{7} \to \frac{\pi}{7} \quad \frac{3\pi}{7} \quad \frac{5\pi}{7} \quad \pi\end{cases}$$

$$\boxed{\max A=\pi}$$

۱۰

$$\tan 3x=\cot x=\tan(\frac{\pi}{2}-x)$$

$$3x=2k\pi+\frac{\pi}{2}-x \to x=\frac{k\pi}{2}+\frac{\pi}{8} \longrightarrow \frac{9\pi}{8} \quad \frac{13\pi}{8}$$

$$3x=2k\pi-\frac{\pi}{2}+x \to x=k\pi-\frac{\pi}{4} \longrightarrow \frac{7\pi}{4}$$

$$\frac{9\pi}{8}+\frac{13\pi}{8}+\frac{14\pi}{8}=\frac{36\pi}{8}=\boxed{\frac{9\pi}{2}}$$