«علیرضا آقاجانی»

①

$$8\cos x - \tan^2 x = 1 \to 8\cos x = 1 + \underbrace{\tan^2 x}_{\frac{1}{\cos^2 x}} \to 8\cos^3 x = 1 \to \cos x = \sqrt[3]{\frac{1}{8}}$$

$$\to \cos x = \frac{1}{2} \to \cos x = \cos\frac{\pi}{3} \to \boxed{x = 2k\pi \pm \frac{\pi}{3}} \to \frac{\pi}{3}, \frac{5\pi}{3} \to$$ ۲ جواب

②

$$5\sin^2 x + 2\cos 3x = -2 \xrightarrow[\sin^2 x = 1-\cos^2 x]{\cos 3x = 4\cos^3 x - 3\cos x} 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$$

$$\to 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x + 2 = 0 \to 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \to$$ جمع ضرایب توان‌های زوج = جمع ضرایب توان‌های فرد

$\boxed{\text{یکی از ریشه‌ها } -1 =}$

$$\begin{array}{l|l} 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 & \cos x + 1 \\ \hline 8\cos^3 x + 8\cos^2 x & \boxed{8\cos^2 x - 13\cos x + 7} \\ -13\cos^2 x - 6\cos x & \\ -13\cos^2 x - 13\cos x & \\ 7\cos x + 7 & \\ 7\cos x + 7 & \\ 0 & \end{array}$$

$$\to (\cos x + 1)\underbrace{(8\cos^2 x - 13\cos x + 7)}_{\Delta < 0} = 0$$

$$\boxed{\cos x = -1} \to \boxed{2k\pi + \pi} \to (-\pi, \pi)$$ تعداد جواب: ۲ تا

③

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$2\sin x \cos 2x + \sin x = 1 \to \sin x(2\cos 2x + 1) = 1 \to \sin x(2 - 4\sin^2 x + 1) = 1$$

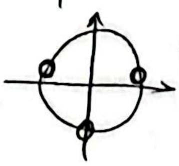
$$\to -4\sin^3 x + 3\sin x - 1 = 0 \to$$ جمع ضرایب توان زوج = جمع ضرایب توان فرد → برای $\sin x = -1$ جواب است، بنابراین

$$\begin{array}{l|l} -4\sin^3 x + 3\sin x - 1 & \sin x + 1 \\ \hline -4\sin^3 x - 4\sin^2 x & -4\sin^2 x + 4\sin x - 1 \\ 4\sin^2 x + 3\sin x & \\ 4\sin^2 x + 4\sin x & \\ -\sin x - 1 & \\ -\sin x - 1 & \\ 0 & \end{array}$$

$$\to (\sin x + 1)\underbrace{(-4\sin^2 x + 4\sin x - 1)}_{(-(2\sin x - 1)^2)} = 0$$

$$2k\pi + \frac{3\pi}{2} \leftarrow \sin x = -1$$

$$2k\pi + \frac{5\pi}{6} \,/\, 2k\pi + \frac{\pi}{6} \leftarrow \sin x = \frac{1}{2}$$



دارای ۳ جواب

④

$$\frac{1}{\underbrace{\sin(\frac{\pi}{2} + 2x)}_{\cos 2x}} + \frac{1}{\underbrace{\cos(\frac{\pi}{2} + 4x)}_{-\sin 4x}} \xrightarrow{\text{مخرج مشترک}} \frac{\sin 4x - \cos 2x}{(\cos 2x)(-\sin 4x)} = 0 \to \sin 4x = \cos 2x$$

$$\to \sin 4x = \sin(\frac{\pi}{2} + 2x)$$

مخرج را صفر می‌کند ← غ‌ق‌ق $x = k\pi + \frac{\pi}{4} \leftarrow 4x = 2k\pi + \frac{\pi}{2} + 2x$

$$x = \frac{k\pi}{3} + \frac{\pi}{12} \leftarrow 4x = 2k\pi + \pi - \frac{\pi}{2} - 2x$$

$$\to \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4} \to$$ تفاضل جواب‌ها $$\to \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3} \xrightarrow{2\times} \tan\frac{2\pi}{3} = \frac{\overbrace{\sin\frac{2\pi}{3}}^{\frac{\sqrt{3}}{2}}}{\underbrace{\cos\frac{2\pi}{3}}_{-\frac{1}{2}}} = -\sqrt{3}$$

غ‌ق‌ق — مخرج را صفر می‌کند

$m(\underbrace{\cos x-\sin x}) - 3\sqrt6\,\sin 2x = \sqrt6 \to m\left(-1\sqrt2\,\sin(x+\frac{3\pi}{4})\right) - 3\sqrt6\,\sin2x = \sqrt6$ (5)

$\frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha)$

$\alpha \Rightarrow \tan\alpha = \frac{b}{a}$

$\to -\sqrt2 m\sin(x+\frac{3\pi}{4}) - 3\sqrt6\,\sin2x = \sqrt6 \xrightarrow[\div -\sqrt2]{} m\sin(x+\frac{3\pi}{4}) + 3\sqrt3\,\sin2x = -\sqrt3$

$\xrightarrow[\underbrace{-x-\frac{\pi}{4},\ x+\frac{3\pi}{4}}_{\text{متمم}}]{\cos(-x-\frac{\pi}{4})=\frac{1}{\sqrt3}} \frac{\sqrt3}{3}m + 3\sqrt3\,\sin2x = -\sqrt3 \to \frac13 m + 3\sin2x = -1 \to \frac13 m = -1 - 3\sin2x$

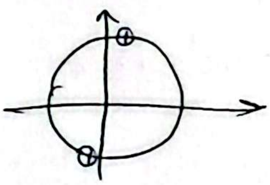
$\to m = -9\sin2x - 3$

(6)

$\cos(x-\frac{\pi}{3}) \to \cos(\overset{\frac{2\pi}{6}}{\cancel{\frac{\pi}{3}}} - x) \to \sin(\frac{\pi}{6}+x)$

$\to \sin^2(\frac{\pi}{6}+x) = 1$

$\begin{cases} \sin(x+\frac{\pi}{6}) = 1 \to x+\frac{\pi}{6} = 2k\pi+\frac{\pi}{2} \to \boxed{x = 2k\pi+\frac{\pi}{3}} \\ \sin(x+\frac{\pi}{6}) = -1 \to x+\frac{\pi}{6} = 2k\pi+\frac{3\pi}{2} \to \boxed{x = 2k\pi+\frac{4\pi}{3}} \end{cases}$



$\to$ تعداد جواب : ۲ تا

(7)

$a\sin x + b\cos x \to \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha)$, $\tan\alpha = \frac{b}{a}$

$\to \sin x + \sqrt3\cos x \to \underset{2}{\cancel{\sqrt4}}\,\sin(x+\frac{\pi}{3}) \Rightarrow 2\sin(x+\frac{\pi}{3}) = \sqrt2$

$\to \sin(x+\frac{\pi}{3}) = \sin\frac{\pi}{4} \to x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \to x = 2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4} \to x = 2k\pi+\frac{2\pi}{3}$

$\to -\frac{\pi}{12},\ \overset{\frac{8\pi}{12}}{\cancel{\frac{2\pi}{3}}},\ \frac{23\pi}{12} \xrightarrow{\text{مجموع}} \frac{15\pi}{6} = \frac{5\pi}{2}$

(8)

$4\sin x\,\underbrace{\sin(\frac{3\pi}{2}-x)}_{-\cos x} = 1 \to -4\sin x\cos x = 1 \to -2\sin2x = 1 \to \sin2x = -\frac12$

مجموع : $\frac{50\pi}{12} \to \frac{15\pi}{6}$

$\to \sin2x = \sin\frac{7\pi}{6} \to 2x = 2k\pi+\frac{7\pi}{6} \to x = k\pi+\frac{7\pi}{12} \to \frac{7\pi}{12},\ \frac{19\pi}{12}$

$2x = 2k\pi-\frac{\pi}{6} \to x = k\pi-\frac{\pi}{12} \to \frac{23\pi}{12},\ \frac{11\pi}{12}$

(۹)

فرمول طلایی → $\cos^2 x = \frac{1+\cos 2x}{2}$

$\longrightarrow$ $1+\cos 2\alpha \to 2\cos^2\alpha$

$1+\cos 4\alpha \to 2\cos^2 2\alpha$

$1+\cos 8\alpha \to 2\cos^2 4\alpha$

$\Big\}\ \underset{\text{(ضرب)}}{\times} \to 8(\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha)^2=\frac{1}{8}$

$\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha = t \to \frac{\sin\alpha}{\sin\alpha}\times t \to \frac{1}{8}\frac{\sin 8\alpha}{\sin\alpha}$

$\to 8\left(\frac{1}{8}\frac{\sin 8\alpha}{\sin\alpha}\right)^2=\frac{1}{8} \Rightarrow \frac{\sin^2 8\alpha}{\sin^2\alpha}=1 \to \frac{\sin 8\alpha}{\sin\alpha}=\pm 1$

$\to \sin 8\alpha = 1\sin\alpha \to 8\alpha = 2\cdot k\pi+\alpha \to \alpha=\frac{2k\pi}{7}$

$8\alpha = 2k\pi+\pi-\alpha \longrightarrow \alpha=\frac{2k\pi}{9}$

$\sin 8\alpha = \underbrace{-\sin\alpha}_{\sin-\alpha}$ $\to 8\alpha = 2k\pi-\alpha \to \alpha=\frac{2k\pi}{9}$

$8\alpha = 2k\pi+\pi+\alpha \to \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$

$\longrightarrow$ $\boxed{\text{بزرگترین} = \frac{8\pi}{9}}$

$\boxed{\tan x=\frac{1}{\cot x}}$ (۱۰)

$\tan 3x = \cot x \quad \tan\left(\frac{\pi}{2}+x\right)$

$3x = k\pi+\frac{\pi}{2}+x \to \boxed{x=\frac{k\pi}{2}+\frac{\pi}{4}} \to \frac{5\pi}{4}, \frac{7\pi}{4} \xrightarrow[\text{جمع}]{} 3\pi$