

$$1 + \tan^2 x = \frac{1}{\cos^2 x} = 1 \cos x \rightarrow 1 = 1 \cos^2 x \rightarrow \cos x = \frac{1}{\sqrt{2}} = \cos \frac{\pi}{4}$$

$$\rightarrow x = 2k\pi \pm \frac{\pi}{4} \rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4} \rightarrow \text{جواب}$$

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$$5(1 - \cos^2 x) + 2 \cos^2 x + 2 = 0 \rightarrow 1 \cos^2 x - 5 \cos^2 x - 2 \cos x + 7 = 0$$

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برابر

$$(x \cos x + 1)(1 \cos^2 x - 1 \cos x + 7) = 0 \rightarrow \cos x = -1 = \cos \pi$$

$\Delta < 0$

$$\rightarrow x = 2k\pi + \pi \rightarrow x = -\pi, \pi \rightarrow \text{جواب}$$

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$$2 \sin x (1 - 2 \sin^2 x) + \sin x = 1 \rightarrow \cancel{2 \sin x} - \cancel{2 \sin^2 x} + \cancel{\sin x} = 1 = \sin^2 x = \sin^2 \frac{\pi}{4}$$

$\sqrt{2} \sin x$

$$\rightarrow 2x = 2k\pi + \frac{\pi}{4} \rightarrow x = \frac{2k\pi}{2} + \frac{\pi}{8}$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}$$

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$$\frac{1}{\sin(\frac{\pi}{4} - 2x)} + \frac{1}{\cos(\frac{\pi}{4} + 2x)} = \frac{1}{\cos 2x} - \frac{1}{\sin 2x} = 0 \rightarrow \frac{1}{\cos 2x} = \frac{1}{2 \sin 2x \cos 2x}$$

$$\rightarrow 1 = \frac{1}{2 \sin 2x} \rightarrow \sin 2x = \frac{1}{2} = \sin \frac{\pi}{6} \rightarrow \begin{cases} 2x = 2k\pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12} \\ 2x = 2k\pi + \pi - \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{2} - \frac{\pi}{12} \end{cases}$$

$$\rightarrow \begin{cases} x_1 = \frac{\pi}{12} \\ x_2 = \frac{5\pi}{12} \end{cases} \rightarrow x_2 - x_1 = \frac{4\pi}{12} = \frac{\pi}{3} \rightarrow \alpha = \frac{\pi}{3} \rightarrow \tan \frac{\pi}{3} = \sqrt{3}$$

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$$\cos(x + \frac{\pi}{2}) = \frac{1}{\sqrt{2}} (\cos x - \sin x) = \frac{1}{\sqrt{2}} \rightarrow \cos x - \sin x = \frac{\sqrt{2}}{\sqrt{2}}$$

توان

$$1 - \sin 2x = \frac{\sqrt{2}}{\sqrt{2}} \rightarrow \sin 2x = \frac{1}{\sqrt{2}}$$

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$$\frac{m\sqrt{4}}{3} - \frac{4\sqrt{4}}{3} = \sqrt{4} \rightarrow \frac{m}{3} - 1 = 1 \rightarrow m = 4$$

$$\sin\left(n + \frac{\pi}{4}\right) \cos\left(\frac{\pi}{4} - n\right) = \sin^2\left(n + \frac{\pi}{4}\right) = 1 \rightarrow \begin{cases} \sin\left(n + \frac{\pi}{4}\right) = 1 = \sin \frac{\pi}{4} \\ \sin\left(n + \frac{\pi}{4}\right) = -1 = \sin \frac{5\pi}{4} \end{cases}$$

$$\rightarrow n + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \rightarrow n = 2k\pi$$

$$\rightarrow n + \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4} \rightarrow n = 2k\pi + \pi \rightarrow n = (2k+1)\pi$$

$$\sin n + \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} \cos n = \sqrt{2} \rightarrow \frac{\sin n \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \cos n}{\cos \frac{\pi}{4}} = \sqrt{2}$$

$$\sin\left(n + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \sin \frac{\pi}{4} \rightarrow \begin{cases} n + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \rightarrow n = 2k\pi \\ n + \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{4} \rightarrow n = 2k\pi + \pi - \frac{\pi}{2} \end{cases}$$

$$n = -\frac{\pi}{4}, \frac{7\pi}{4} \rightarrow \frac{7\pi}{4} = \frac{11\pi}{4}$$

$$-\sqrt{2} \sin n \cos n = -\sqrt{2} \sin 2n = 1 \rightarrow \sin 2n = -\frac{1}{\sqrt{2}} = \sin\left(-\frac{\pi}{4}\right)$$

$$\begin{cases} 2n = 2k\pi + \frac{\pi}{4} \rightarrow n = \frac{(4k+1)\pi}{8} \\ 2n = 2k\pi + \pi - \frac{\pi}{4} \rightarrow n = \frac{(4k+3)\pi}{8} \end{cases}$$

$$\begin{cases} 2n = 2k\pi + \frac{\pi}{4} \rightarrow n = \frac{(4k+1)\pi}{8} \\ 2n = 2k\pi + \pi - \frac{\pi}{4} \rightarrow n = \frac{(4k+3)\pi}{8} \end{cases}$$

$$\sqrt{2} \cos^3 \alpha \times \sqrt{2} \cos^3 \alpha \times \sqrt{2} \cos^3 \alpha = 1 \frac{\sin^3 \alpha}{\sin^3 \alpha} \cos^3 \alpha \cos^3 \alpha \cos^3 \alpha$$

$$= \sqrt{2} \frac{\sin^3 \alpha}{\sin^3 \alpha} \cos^3 \alpha \cos^3 \alpha \cos^3 \alpha = \frac{1}{\sqrt{2}} \frac{\sin^3 \alpha}{\sin^3 \alpha} \cos^3 \alpha \cos^3 \alpha \cos^3 \alpha = \frac{1}{\sqrt{2}} \frac{\sin^3 \alpha}{\sin^3 \alpha} = \frac{1}{\sqrt{2}}$$

$$\rightarrow \begin{cases} \sin 3\alpha = \sin \alpha \rightarrow \begin{cases} 3\alpha = 2k\pi + \alpha \rightarrow 2\alpha = 2k\pi \rightarrow \alpha = k\pi \\ 3\alpha = 2k\pi + \pi - \alpha \rightarrow 4\alpha = 2k\pi + \pi \rightarrow \alpha = \frac{(2k+1)\pi}{4} \end{cases} \\ \sin 3\alpha = -\sin \alpha = \sin(-\alpha) \rightarrow \begin{cases} 3\alpha = 2k\pi - \alpha \rightarrow 4\alpha = 2k\pi \rightarrow \alpha = k\pi \\ 3\alpha = 2k\pi + \pi + \alpha \rightarrow 2\alpha = 2k\pi + \pi \rightarrow \alpha = \frac{(2k+1)\pi}{2} \end{cases} \end{cases}$$

$$\alpha = \frac{\pi}{2} \checkmark$$

$$\tan 3n = \cot n = \tan\left(\frac{\pi}{2} - n\right) \rightarrow 3n = k\pi + \left(\frac{\pi}{2} - n\right) \rightarrow 4n = k\pi + \frac{\pi}{2} \rightarrow n = k\pi + \frac{\pi}{8}$$

$$\rightarrow n = k\frac{\pi}{8} + \frac{\pi}{8} \rightarrow n = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$