نام و نام خانوادگی : حسام رفیع زاده   «به نام خدا»   مقطع: دوازدهم پسر A

شماره: ۱۷

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$$8\cos x-\tan^2 x=1 \rightarrow 8\cos x=1+\tan^2 x \rightarrow 8\cos=\frac{1}{\cos^2 x} \rightarrow \cos^3 x=\frac{1}{8} \rightarrow \cos x=\pm\frac{1}{2}$$

$\rightarrow x=2k\pi\pm\frac{\pi}{3}$  $\rightarrow$ در بازه $[0,2\pi]$ دو جواب

۲-

$$5\sin^2 x+2\cos 3x=-2 \rightarrow 5\sin^2 x+8\cos^3 x-6\cos x=-2$$

$$\rightarrow 8\cos^3 x-6\cos x-5\cos^2 x+7=0 \rightarrow \begin{cases}\cos x=-1\\ \Delta<0\end{cases}$$

$\rightarrow \cos x=-1 \rightarrow$  $[-\pi,\pi]$

این معادله دارای دو جواب می باشد

۳-

$$2\sin x\cdot\cos 2x+\sin x=1 \rightarrow \sin x(2\cos 2x+1)=1$$

$$\rightarrow \sin x\left(2(\cos^2 x-\sin^2 x)+\sin^2 x+\cos^2 x\right)=1 \rightarrow \sin x(3\cos^2 x-\sin^2 x)=1$$

$$\xrightarrow{\cos^2 x=1-\sin^2 x} \sin x(3-4\sin^2 x)=1 \rightarrow 3\sin x-4\sin^3 x-1=0$$

$$\rightarrow 4\sin^3 x-3\sin x+1=0 \begin{cases}\sin x=-1\\ \sin x=\frac{1}{2}\end{cases}$$

$$\frac{\pi}{6}+\pi-\frac{\pi}{6}+\frac{3\pi}{2}=\pi+\frac{3\pi}{2}=\frac{5\pi}{2}$$

-۴

$$\frac{1}{\sin(\frac{\pi}{2}+2x)}+\frac{1}{\cos(\frac{\pi}{2}+4x)}=0 \rightarrow \frac{1}{\cos 2x}+\frac{1}{-\sin 4x}=0 \rightarrow \frac{\sin 4x-\cos 2x}{\sin 4x\cdot\cos 2x}=0$$

$$\rightarrow \underset{①}{\sin 4x-\cos 2x=0}\ ,\ \underset{②}{\sin 4x\cdot\cos 2x\neq 0}$$

$$\xrightarrow{①} \sin 4x=\cos 2x \rightarrow \sin 4x=\sin(\pi/2+2x) \rightarrow 4x=2k\pi+\pi/2+2x \rightarrow x=k\pi+\pi/4$$

$$\rightarrow 4x=2k\pi+\pi-\pi/2-2x \rightarrow x=\frac{k\pi}{3}+\pi/12$$

$$\xrightarrow{②} \cos 2x\neq 0 \rightarrow 2x\neq k\pi+\pi/2 \rightarrow x\neq k\pi/2+\pi/4$$

و

$$\sin 4x\neq 0 \rightarrow 4x\neq k\pi \rightarrow x\neq\frac{k\pi}{4}$$

$$\xrightarrow{①\cap②} \underset{\text{به جز } \frac{k\pi}{4} \text{ ها}}{x=\frac{k\pi}{3}+\pi/12} \xrightarrow[{[0,\pi]}]{\text{در بازه}} \underset{\text{جوابها}}{x=\frac{\pi}{12},\frac{5\pi}{12}} \rightarrow \text{اختلاف جوابها}\quad \alpha=\frac{4\pi}{12}=\frac{\pi}{3}$$

$$\tan\frac{2\pi}{3}=\tan(\pi-\pi/3)=-\tan\frac{\pi}{3}=-\sqrt{3}$$

-۵

$$\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{3}} \rightarrow \frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}} \rightarrow \cos x-\sin x=\frac{2}{\sqrt{6}}\ ①$$

$$\xrightarrow{(\ )^2} \underbrace{\cos^2 x+\sin^2 x}_{1}-2\cos x\cdot\sin x=\frac{1}{3} \rightarrow \sin 2x=\frac{2}{3}\ ②$$

$$m(\cos x-\sin x)-3\sqrt{6}\sin 2x=\sqrt{6} \xrightarrow{①,②} m(\frac{2}{\sqrt{6}})-\cancel{3}\sqrt{6}\times\frac{2}{\cancel{3}}=\sqrt{6}$$

$$\xrightarrow{\times\sqrt{6}} 2m-12=6 \rightarrow 2m=18 \rightarrow \boxed{m=9}$$

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-۶

$$\sin\left(x+\frac{\pi}{6}\right)\cdot\cos\left(x-\frac{\pi}{3}\right)=1 \longrightarrow \left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)^2=1$$

$$\longrightarrow \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\pm 1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1 \longrightarrow \begin{cases} x+\pi/6 = 2k\pi+\pi/2 \\ x+\pi/6 = 2k\pi-\pi/2 \end{cases}$$

$$\longrightarrow x = 2k\pi+\pi/3$$
$$x = 2k\pi - 2\pi/3$$

$\boxed{\text{دو جواب}}$

-۷

$$\sin x+\sqrt{3}\cos x=\sqrt{2} \longrightarrow 2\left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)=\sqrt{2}$$

$$\longrightarrow \sin\left(x+\frac{\pi}{3}\right)=\sqrt{2}/2 \longrightarrow \sin\left(x+\frac{\pi}{3}\right)=\sin\frac{\pi}{4} \longrightarrow \begin{cases} x+\frac{\pi}{3} = 2k\pi+\pi/4 \\ x+\pi/3 = 2k\pi+\pi-\pi/4 \end{cases}$$

$$\longrightarrow x = 2k\pi-\pi/12 \xrightarrow{k=0,1} \frac{-\pi}{12},\ \frac{23\pi}{12}$$

$$\longrightarrow x = 2k\pi+\frac{5\pi}{12} \xrightarrow{k=0} \frac{5\pi}{12} \qquad \xrightarrow{\text{جمع}} \frac{23\pi}{12}+\frac{5\pi}{12}=\boxed{\frac{27\pi}{12}}$$

-۸

$$4\sin x\cdot\sin\left(\frac{3\pi}{2}-x\right)=1 \longrightarrow 4\sin x\cdot\underbrace{\underbrace{\sin(\pi+\pi/2-x)}_{-\sin(\pi/2-x)}}_{-\cos x}=1$$

$$\longrightarrow 2\sin x\cdot\cos x=-1/2$$

$$\longrightarrow \sin 2x=-1/2 \longrightarrow 2x=2k\pi-\pi/6 \longrightarrow x=k\pi-\pi/12 \xrightarrow{k=1,2} \pi-\pi/12,\ 2\pi-\pi/12$$

$$\longrightarrow 2x=2k\pi+\pi+\pi/6 \longrightarrow x=k\pi+7\pi/12 \xrightarrow{k=0,1} \frac{7\pi}{12},\ \pi+\frac{7\pi}{12}$$

$$\xrightarrow{\text{جمع}} 4\pi+\frac{14\pi}{12}-\frac{2\pi}{12}=\boxed{5\pi}$$

9-

$$(\sin^2\alpha+\cos^2\alpha+\cos^2\alpha-\sin^2\alpha)(\sin^2 2\alpha+\cos^2 2\alpha+\cos^2 2\alpha-\sin^2 2\alpha)(\sin^2 4\alpha+\cos^2 4\alpha+\cos^2 4\alpha-\sin^2 4\alpha)=\frac{1}{8}$$

$$\rightarrow 2\cos^2\alpha \times 2\cos^2 2\alpha \times 2\cos^2 4\alpha = \frac{1}{8} \rightarrow \cos^2\alpha \times \cos^2 2\alpha \times \cos^2 4\alpha = \frac{1}{64}$$

$$\xrightarrow{\times \sin^2\alpha} \frac{1}{4}\underbrace{\underbrace{\sin^2 2\alpha \times \cos^2 2\alpha}_{1/4 \sin^2 4\alpha} \times \cos^2 4\alpha}_{1/4 \sin^2 8\alpha} = \frac{1}{64}\times \sin^2\alpha \rightarrow \frac{1}{\cancel{64}}\sin^2 8\alpha = \frac{1}{\cancel{64}}\times\sin^2\alpha$$

$$\rightarrow \sin^2 8\alpha = \sin^2\alpha \rightarrow \sin 8\alpha = \sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi+\alpha \end{cases}$$

$$\rightarrow \sin 8\alpha = -\sin\alpha \rightarrow \sin(8\alpha) = \sin(\pi+\alpha) \rightarrow \begin{cases} 8\alpha = 2k\pi+\cancel{\pi}-\cancel{\pi}-\alpha \\ 8\alpha = 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{9}+\pi/9$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\text{MAX}} \boxed{\frac{4\pi}{9}} \qquad A:\left\{0, \pi/9, 3\pi/9, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{\pi}{7}, \frac{3\pi}{7}, \frac{2\pi}{7}\right\}$$

$$\alpha = \frac{2k\pi}{7}$$

$$\alpha = \frac{2k\pi}{7}+\frac{\pi}{7}$$

10-

$$\tan(3x).\tan x = 1 \rightarrow \frac{\sin 3x.\sin x}{\cos 3x.\cos x} = 1 \rightarrow \sin 3x.\sin x = \cos 3x.\cos x$$

$$\rightarrow \cos 3x.\cos x - \sin 3x.\sin x = 0 \rightarrow \cos 4x = 0 \rightarrow 4x = 2k\pi \pm \pi/2$$

$$\rightarrow x = \frac{k\pi}{2} \pm \frac{\pi}{8} \xrightarrow[{[\pi, 2\pi]}]{\text{در بازه}} \begin{array}{l} x = \frac{k\pi}{2}+\frac{\pi}{8} \xrightarrow{k=2,3} \pi+\frac{\pi}{8}, \frac{13\pi}{8} \\ x = \frac{k\pi}{2}-\frac{\pi}{8} \xrightarrow{k=3} \frac{11\pi}{8}, 2\pi-\frac{\pi}{8} \end{array}$$

$$\xrightarrow[\text{دامنه}]{\text{چک کردن}} \cos 3x, \cos x \neq 0 \xrightarrow[{[\pi, 2\pi]}]{} x = \frac{9\pi}{8}, \frac{13\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$$

$$\xrightarrow[\text{جواب}]{\text{مجموع}} \frac{48\pi}{8} = \boxed{6\pi}$$