(1) $8\cos\alpha - \tan^2\alpha = 1 \Rightarrow 8\cos\alpha = 1 + \tan^2\alpha \Rightarrow 8\cos\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 8\cos^3\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$ یا $\frac{5\pi}{3}$

(2) $5\sin^2\alpha + 2\cos^2 2\alpha = -2 \Rightarrow 5(1-\cos^2\alpha) + 2(4\cos^4\alpha - 4\cos\alpha) = -2 \Rightarrow$

$\Rightarrow 5 - 5\cos^2\alpha + 8\cos^4\alpha - 8\cos\alpha + 2 = 0 \Rightarrow 8\cos^4\alpha - 5\cos^2\alpha - 8\cos\alpha + 7 = 0 \Rightarrow 8-8 = 7-5 \Rightarrow$

$\cos\alpha = -1 \quad \alpha = \pi$

(3) $2\sin\alpha \times \cos 2\alpha + \sin\alpha = 1 \Rightarrow 2\sin\alpha \times (1 - 2\sin^2\alpha) + \sin\alpha = 1 \Rightarrow 2\sin\alpha - 4\sin^3\alpha + \sin\alpha = 1 \Rightarrow$

$\Rightarrow 4\sin^3\alpha - 3\sin\alpha + 1 = 0 \Rightarrow \sin\alpha = \{-1\} \cup \{\frac{1}{2}\} \quad \alpha = \frac{\pi}{6}$ یا $\frac{5\pi}{6}$ یا $\frac{3\pi}{2} \quad \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$

(4) $\frac{1}{\sin\left(\frac{\pi + 4\alpha}{2}\right)} + \frac{1}{\cos\left(\frac{\pi + 8\alpha}{2}\right)} = 0 \Rightarrow \frac{1}{\sin\left(\frac{\pi}{2} + 2\alpha\right)} + \frac{1}{\cos\left(\frac{\pi}{2} + 4\alpha\right)} = 0 \Rightarrow \frac{1}{\cos 2\alpha} - \frac{1}{\sin 4\alpha} = 0 \Rightarrow$

$\Rightarrow \frac{\sin 4\alpha - \cos 2\alpha}{\sin 4\alpha \cos 4\alpha} = 0 \Rightarrow \sin 4\alpha - \cos 2\alpha = 0 \Rightarrow \sin 4\alpha = \cos 2\alpha \Rightarrow 2\sin 2\alpha \cos 2\alpha = \cos 2\alpha \Rightarrow$

$2\sin 2\alpha \cos 2\alpha - \cos 2\alpha = 0 \Rightarrow \cos 2\alpha(2\sin 2\alpha - 1) = 0 \Rightarrow \begin{cases} \cos 2\alpha = 0 \\ 2\sin 2\alpha - 1 = 0 \Rightarrow \sin 2\alpha = \frac{1}{2} \end{cases}$

$\begin{cases} 2\alpha = 2k\pi + \frac{\pi}{6} \Rightarrow \alpha = k\pi + \frac{\pi}{12} \\ 2\alpha = 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6} \Rightarrow \alpha = k\pi + \frac{5\pi}{12} \end{cases}$

$\alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$

$\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$

(5) $\cos\left(\alpha + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{3} \Rightarrow \cos\alpha\cos\frac{\pi}{3} - \sin\alpha\sin\frac{\pi}{3} = \frac{\sqrt{3}}{3} \Rightarrow \frac{\sqrt{3}}{2}(\cos\alpha - \sin\alpha) = \frac{\sqrt{3}}{3} \Rightarrow$

$\cos\alpha - \sin\alpha = \frac{2}{3}\sqrt{\frac{3}{2}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3} \qquad m(\cos\alpha - \sin\alpha) - 4\sqrt{3}\sin 2\alpha = \sqrt{3} \Rightarrow \frac{\sqrt{6}}{3}m - 4\sqrt{3}\times 2\sin\alpha\cos\alpha = \sqrt{3}$

$\Rightarrow \frac{\sqrt{3}}{3}m - 4\sqrt{3} \times \frac{\sqrt{3}}{3} = \sqrt{6} \Rightarrow \frac{\sqrt{6}}{2}m = 2\sqrt{6} \Rightarrow m = 4$

(6) $\sin\left(\alpha + \frac{\pi}{6}\right)\cos\left(\alpha - \frac{\pi}{3}\right) = 1 \Rightarrow$ به دلیل تساوی بودن زاویه‌ها $\Rightarrow \sin^2\left(\alpha + \frac{\pi}{6}\right) = 1 \Rightarrow \sin\left(\alpha + \frac{\pi}{6}\right) = \pm 1 \Rightarrow$

$\alpha + \frac{\pi}{6} = k\pi + \frac{\pi}{2} \Rightarrow \alpha = k\pi + \frac{\pi}{3}$

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(7) $\sin\alpha + \sqrt{3}\cos\alpha = \sqrt{2} \Rightarrow \frac{1}{1}\sqrt{1+3}\ \sin\left(\alpha + \frac{\pi}{3}\right) = 2\sin\left(\alpha + \frac{\pi}{3}\right) = \sqrt{2} \Rightarrow \sin\left(\alpha + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \Rightarrow$

$\begin{cases} \alpha + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \Rightarrow \alpha = 2k\pi - \frac{\pi}{12} \\ \alpha + \frac{\pi}{3} = 2k\pi + \pi - \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \Rightarrow \alpha = 2k\pi + \frac{5\pi}{12} \end{cases}$

$\alpha = \left\{\frac{\pi}{12}\right\} \cup \left\{\frac{5\pi}{12}\right\} \cup \left\{\frac{23\pi}{12}\right\}$

$-\frac{\pi}{12} + \frac{5\pi}{12} + \frac{23\pi}{12} = \frac{27\pi}{12}$

(8) $4\sin\alpha\sin\left(\frac{3\pi}{2} - \alpha\right) = 1 \Rightarrow 4(\sin\alpha)(-\cos\alpha) = 1 \Rightarrow -4\sin\alpha\cos\alpha = 1 \Rightarrow -2\sin 2\alpha = 1 \Rightarrow \sin 2\alpha = -\frac{1}{2}$

$\begin{cases} 2\alpha = 2k\pi - \frac{\pi}{6} \Rightarrow \alpha = k\pi - \frac{\pi}{12} \\ 2\alpha = 2k\pi - \pi + \frac{\pi}{6} \Rightarrow \alpha = k\pi - \frac{5\pi}{12} \end{cases}$

$\frac{\sqrt{\pi}}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi$

(9) $(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8} \Rightarrow 2\cos^2\alpha\ 2\cos^2 2\alpha\ 2\cos^2 4\alpha = \frac{1}{8} \Rightarrow \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \Rightarrow$

$\frac{\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha}{\sin\alpha} = \pm\frac{1}{8} \Rightarrow \sin 8\alpha = \pm\sin\alpha \Rightarrow$

$\begin{cases} 8\alpha = 2k\pi + \alpha \\ 8\alpha = 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \\ 8\alpha = 2k\pi - \pi + \alpha \end{cases}$

$A = \left\{\frac{4\pi}{7}\right\} \cup \left\{\frac{\sqrt{\pi}}{9}\right\} \cup \left\{\frac{\Lambda\pi}{9}\right\} \cup \left\{\frac{5\pi}{7}\right\}$

$\frac{\Lambda\pi}{9}$ بیشترین مقدار جواب

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$\tan 3\alpha \tan\alpha = 1 \Rightarrow \tan^2\alpha = \frac{1}{\tan\alpha} \Rightarrow \tan^2\alpha = \cot\alpha \Rightarrow$

$\tan 3\alpha = \tan\left(\frac{\pi}{2} - \alpha\right) \Rightarrow 3\alpha = k\pi + \frac{\pi}{2} - \alpha \Rightarrow \alpha = \frac{k\pi}{4} + \frac{\pi}{8}$

$\alpha = \left\{ \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \right\}$

$\frac{9\pi}{8} + \frac{11\pi}{8} + \frac{13\pi}{8} + \frac{15\pi}{8} = \frac{48\pi}{8} = 4\pi$

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