

الف)  $\lim_{x \rightarrow 2^+} \frac{x+9}{x+1} = \frac{2+9}{2+1} = \frac{11}{3} = 3\frac{2}{3}$  ✓

ب)  $\lim_{x \rightarrow 2^-} \frac{x+9}{x+1} = \frac{2+9}{2+1} = \frac{11}{3} = 3\frac{2}{3}$  ✓

ج)  $\lim_{x \rightarrow 2} \frac{x+9}{x+1} = 3\frac{2}{3}$  ✓

د)  $\lim_{x \rightarrow 0} \frac{2x+7}{[x]^2} = \frac{0+7}{[0]^2} = \frac{7}{0^+} = +\infty$  ✓

الف)  $\lim_{x \rightarrow 3^-} f(x) - 2 = f(3^-) - 2 = 1 - 2 = -1$  ✓

ب)  $\lim_{x \rightarrow 3^-} [f(x) - 2] = \lim_{x \rightarrow 3^-} [f(3^-) - 2] = \lim_{x \rightarrow 3^-} [1 - 2] = -1$  ✓

ج)  $\left[ \lim_{x \rightarrow 3^-} f(x) - 2 \right] = \left[ \lim_{x \rightarrow 3^-} 1 - 2 \right] = [1 - 2] = [-1] = -1$  ✓

د)  $\left[ \lim_{x \rightarrow 3^+} f(x) - 2 \right] = \left[ \lim_{x \rightarrow 3^+} 1 - 2 \right] = [1 - 2] = [-1] = -1$  ✓

الف)  $\lim_{x \rightarrow 2^-} \frac{3[x]+1}{x} = \frac{3[2^-]+1}{2} = \frac{3 \cdot 1 + 1}{2} = \frac{4}{2} = 2$  ✓

ب)  $\lim_{x \rightarrow 2^-} \left[ \frac{3x+1}{x} \right] = \left[ \frac{3 \cdot 2 + 1}{2} \right] = \left[ \frac{7}{2} \right] = 3\frac{1}{2}$  ✓

ج)  $\left[ \lim_{x \rightarrow 2^-} \frac{3x+1}{x} \right] = \left[ \lim_{x \rightarrow 2^-} \frac{7}{2} \right] = \left[ \frac{7}{2} \right] = 3\frac{1}{2}$  ✓

د)  $\left[ \lim_{x \rightarrow 2^+} \frac{3x+1}{x} \right] = \left[ \lim_{x \rightarrow 2^+} \frac{7}{2} \right] = \left[ \frac{7}{2} \right] = 3\frac{1}{2}$  ✓

الف)  $\lim_{x \rightarrow 1} \frac{x-9}{x^2-9x+9} = \lim_{x \rightarrow 1} \frac{x-9}{(x-9)(x-1)} = \lim_{x \rightarrow 1} \frac{1}{x-1} = \frac{1}{0^+} = +\infty$  ✓  
عدد ندارد

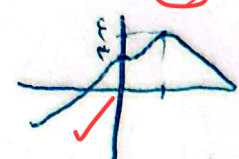
ب)  $\lim_{x \rightarrow 1} \frac{x-9}{x^2-9x+9} = \lim_{x \rightarrow 1} \frac{x-9}{(x-3)^2}$   
 $\begin{matrix} 1^+ & \frac{1-9}{(1^+-3)^2} = \frac{-8}{4} = -2 \\ 1^- & \frac{1-9}{(1^--3)^2} = \frac{-8}{4} = -2 \end{matrix}$  ✓  
عدد ندارد

الف)  $\lim_{x \rightarrow 3} \left[ \frac{-x^2}{9} \right] - \left[ \frac{9x}{10} \right]$   
 $\begin{matrix} 3^+ & = \left[ \frac{-9}{9} \right] - \left[ \frac{27}{10} \right] = -1 - 2\frac{7}{10} = -3\frac{7}{10} \\ 3^- & = \left[ \frac{-9}{9} \right] - \left[ \frac{27}{10} \right] = -1 - 2\frac{7}{10} = -3\frac{7}{10} \end{matrix}$  ✓

ب)  $\lim_{x \rightarrow \frac{1}{14}} \left[ \frac{-1}{x^2} \right]$   
 $\left( -\frac{1}{14} \right)^- \rightarrow \frac{-1}{\frac{1}{196}} = -196$   
 $\left( -\frac{1}{14} \right)^+ \rightarrow \frac{-1}{\frac{1}{196}} = -196$   
 $\lim_{x \rightarrow \frac{1}{14}} \left[ \frac{-1}{x^2} \right] = -196$

$x < -\frac{1}{14} \rightarrow x^2 > \frac{1}{196} \rightarrow -x^2 < -\frac{1}{196} \rightarrow -\frac{1}{x^2} > -196$



الف)  $\lim_{x \rightarrow 1} [x^3 - 3x^2 + 3] = [1]$   
 $y' = -12x^3 + 12x^2 \rightarrow x^2(-12x + 12) \stackrel{0}{=} 0$   
 $\rightarrow x = 0, x = 1$   


ب)  $\lim_{x \rightarrow 0} [x^3 - 3x^2 + 3] =$   
 $\lim_{x \rightarrow 0^+} [x^3] = 0$   
 $\lim_{x \rightarrow 0^-} [x^3] = 0$   
 بالتوجه بشكل الف  $\rightarrow = 3 - [0] = 3$

الف)  $\lim_{x \rightarrow 2} \frac{|x^3 - 1|}{x^2 - 5}$   
 $= \lim_{x \rightarrow 2} \frac{(x-1)(x^2+x+1)}{x^2-5}$   
 $\frac{2^3-1}{2^2-5} = \frac{7}{-1} = -7$   
 $\frac{2^3-1}{2^2-5} = \frac{7}{-1} = -7$

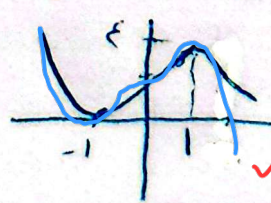
ب)  $\lim_{x \rightarrow \pi} \frac{|\sin x|}{\sin 2x}$   
 $\frac{0}{0}$   $\rightarrow \frac{-\sin x}{\sin 2x} = \frac{-\sin x}{2 \sin x \cos x} = \frac{-1}{2 \cos x}$   
 $\frac{1}{\cos x} = \frac{1}{\cos \pi} = \frac{1}{-1} = -1$   
 $\frac{1}{\cos x} = \frac{1}{\cos \pi} = \frac{1}{-1} = -1$

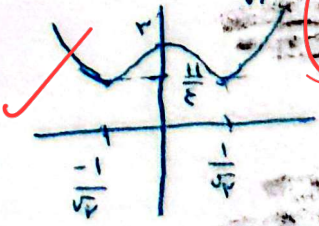
الف)  $\lim_{x \rightarrow \frac{\pi}{2}} \sqrt{\cos x}$   
 $\frac{0}{0}$   $\rightarrow \sqrt{0^+} = 0^+$   
 $\frac{0}{0}$   $\rightarrow \sqrt{0^-} = 0^-$

ب)  $\lim_{x \rightarrow 0} \sqrt{x - 2\sqrt{x}}$   
 $\frac{0}{0}$   $\rightarrow \sqrt{x - 2\sqrt{x}} = \sqrt{x} \sqrt{1 - 2\sqrt{\frac{x}{x}}}$   
 $\sqrt{x} \sqrt{1 - 2} = \sqrt{x} \sqrt{-1}$   
 $\sqrt{x} \sqrt{-1} = \sqrt{x} i$   
 $\sqrt{x} i \rightarrow 0$

الف)  $\lim_{x \rightarrow 0^+} \frac{(-1)^{\lfloor \sin x \rfloor}}{x^3 - x^2}$   
 $= \lim_{x \rightarrow 0^+} \frac{(-1)^1}{0^+} = \frac{-1}{0^+} = -\infty$   
 $\frac{0}{0}$   $\rightarrow \frac{1}{+|-|+}$

ب)  $\lim_{x \rightarrow 2} \frac{x^3 - [x^3]}{x - [x]}$   
 $\frac{0}{0}$   $\rightarrow \frac{x^3 - (x-1)^3}{x - (x-1)} = \frac{x^3 - (x^3 - 3x^2 + 3x - 1)}{1} = 3x^2 - 3x + 1$   
 $\frac{2^3 - 1}{2 - 1} = \frac{7}{1} = 7$

الف)  $y = -3x^3 + 4x^2 + 2$   
 $y' = -9x^2 + 8x \rightarrow x(-9x + 8)$   
 $\rightarrow x = 0, x = \frac{8}{9}$   


ب)  $y = x^3 - x^2 + 2 \rightarrow y' = 3x^2 - 2x$   
 $x(3x - 2) = 0 \rightarrow x = 0, x = \frac{2}{3}$   


الف)  $\lim_{x \rightarrow 0^+} \frac{(-1)^{\lfloor \sin x \rfloor}}{x^3 - x^2} = \lim_{x \rightarrow 0^+} \frac{(-1)^{0^+}}{x^2(x-1)} = \lim_{x \rightarrow 0^+} \frac{1}{0^+} = +\infty$

