

الف) $\lim_{x \rightarrow 2^+} \frac{x+9}{x+1} = \frac{2+9}{2+1} = \frac{11}{3} = 3$

ب) $\lim_{x \rightarrow 2^-} \frac{x+9}{x+1} = \frac{2+9}{2+1} = 3$

ج) $\lim_{x \rightarrow 2} \frac{x+9}{x+1} = 3$

د) $\lim_{x \rightarrow 0} \frac{x^2+9}{x^2} = \frac{0+9}{(0^+)^2} = \frac{9}{0^+} = +\infty$

الف) $\lim_{x \rightarrow 3^-} f(x) - 2 = f(3^-) - 2 = 11 - 2 = 9$

ب) $\lim_{x \rightarrow 3^-} [f(x) - 2] = \lim_{x \rightarrow 3^-} [f(3^-) - 2] = \lim_{x \rightarrow 3^-} [11 - 2] = 9$

ج) $\left[\lim_{x \rightarrow 3^-} f(x) - 2 \right] = \left[\lim_{x \rightarrow 3^-} 11 - 2 \right] = [9] = 9$

د) $\left[\lim_{x \rightarrow 3^+} f(x) - 2 \right] = \left[\lim_{x \rightarrow 3^+} 11 - 2 \right] = [9] = 9$

الف) $\lim_{x \rightarrow 3^-} \frac{x^2[x]+1}{x} = \frac{x^2[3^-]+1}{3} = \frac{10}{3} = 3$

ب) $\lim_{x \rightarrow 3^-} \left[\frac{x^2[x]+1}{x} \right] = \left[\frac{10}{3} \right] = 3$

ج) $\left[\lim_{x \rightarrow 3^-} \frac{x^2[x]+1}{x} \right] = \left[\lim_{x \rightarrow 3^-} 10 \right] = [10] = 10$

د) $\left[\lim_{x \rightarrow 3^+} \frac{x^2[x]+1}{x} \right] = \left[\lim_{x \rightarrow 3^+} 10 \right] = [10] = 10$

الف) $\lim_{x \rightarrow 1} \frac{x-9}{x^2-9x+9} = \lim_{x \rightarrow 1} \frac{x-9}{(x-9)(x-1)} = \lim_{x \rightarrow 1} \frac{1}{x-1} = \frac{1}{0^+} = +\infty$
 $\frac{1}{0^-} = -\infty$
 عدد ندارد

ب) $\lim_{x \rightarrow 1} \frac{x-9}{x^2-9x+9} = \lim_{x \rightarrow 1} \frac{x-9}{(x-9)^2} = \frac{1}{(0^+)^2} = +\infty$
 $\frac{1}{(0^-)^2} = +\infty$
 عدد ندارد

الف) $\lim_{x \rightarrow 3} \left[\frac{x^2}{9} \right] - \left[\frac{9x}{18} \right] = \left[\frac{9}{9} \right] - \left[\frac{27}{18} \right] = 1 - 1.5 = -0.5$
 $\lim_{x \rightarrow 3^+} = \left[\frac{9}{9} \right] - \left[\frac{27}{18} \right] = 1 - 1.5 = -0.5$
 $\lim_{x \rightarrow 3^-} = \left[\frac{9}{9} \right] - \left[\frac{27}{18} \right] = 1 - 1.5 = -0.5$

ب) $\lim_{x \rightarrow \frac{1}{2}} \left[\frac{-1}{x^2} \right] = \left[\frac{-1}{(\frac{1}{2})^2} \right] = \left[\frac{-1}{\frac{1}{4}} \right] = [-4]$
 $\lim_{x \rightarrow \frac{1}{2}^+} = \left[\frac{-1}{(\frac{1}{2})^2} \right] = [-4]$
 $\lim_{x \rightarrow \frac{1}{2}^-} = \left[\frac{-1}{(\frac{1}{2})^2} \right] = [-4]$

الف) $\lim_{x \rightarrow 1} [x^3 - 3x^2 + 3] = [1]$
 $y' = -12x^3 + 12x^2 \rightarrow x^2(-12x + 12) \stackrel{0}{=} 0$
 $\rightarrow x = 0, x = 1$

Sign chart: $\begin{array}{c|c|c|} + & + & - \\ \hline & +3 & +5 \end{array}$

ب) $\lim_{x \rightarrow 0} [x^3 - 3x^2 + 3] = [3]$
 بالتوجه بشكل الف
 $\rightarrow = 3 - [0] = 3$

ج) $\lim_{x \rightarrow 2} \frac{|x^3 - 1|}{x^2 - 5}$
 $= \lim_{x \rightarrow 2} \frac{(x-1)(x^2+x+1)}{x^2-5}$
 $\begin{array}{l} x+ \rightarrow \frac{x^3+x^2+1}{x+2} = \frac{12}{7} = 1.714 \\ x- \rightarrow \frac{x^3+x^2+1}{x+2} = \frac{-12}{7} = -1.714 \end{array}$

د) $\lim_{x \rightarrow \pi} \frac{|\sin x|}{\sin^2 x}$
 $\begin{array}{l} \pi^+ \rightarrow \sin x < 0 \rightarrow -\sin x \\ \pi^- \rightarrow \sin x > 0 \rightarrow \sin x \end{array}$
 $\begin{array}{l} \pi^+ \rightarrow \frac{1}{\cos x} = \frac{1}{\cos \pi^+} = \frac{1}{-1} = -1 \\ \pi^- \rightarrow \frac{1}{\cos x} = \frac{1}{\cos \pi^-} = \frac{1}{-1} = -1 \end{array}$

هـ) $\lim_{x \rightarrow \sqrt{2}} \sqrt{\cos x}$
 $\begin{array}{l} \sqrt{2}^+ \rightarrow \sqrt{0^+} = 0^+ \\ \sqrt{2}^- \rightarrow \sqrt{0^-} = 0^- \end{array}$

و) $\sqrt{x - 2\sqrt{x}}$
 $\begin{array}{l} 0^+ \rightarrow \sqrt{x - 2\sqrt{x}} = \sqrt{x} = 0^+ \\ 0^- \rightarrow \sqrt{x - 2\sqrt{x}} = 0^- \end{array}$

ز) $\lim_{x \rightarrow 0^+} \frac{(-1)^{\lfloor \sin x \rfloor}}{x^3 - x^2}$
 $= \lim_{x \rightarrow 0^+} \frac{(-1)^1}{0^-} = \frac{-1}{0^-} = +\infty$

Sign chart: $\begin{array}{c|c|c|} 0 & 1 & \\ \hline + & - & + \end{array}$

ح) $\lim_{x \rightarrow 2} \frac{x^3 - [x^3]}{x - [x]}$
 $\begin{array}{l} x \rightarrow 2^+ [x] = 2, [x^3] = 8 \rightarrow \frac{x^3 - 8}{x - 2} = x + 2 = 4 \\ x \rightarrow 2^- [x] = 1, [x^3] = 8 \rightarrow \frac{x^3 - 8}{x - 1} = \frac{8 - 8}{1 - 1} = 0 \end{array}$

ط) $y = -3x^3 + 4x^2 + 2$
 $y' = -12x^3 + 8x \rightarrow y' = x^2(-12x + 8)$
 $\rightarrow x = 0, x = \pm \frac{2}{3}$

Sign chart: $\begin{array}{c|c|c|c|} - & 0 & + & + \\ \hline & -1 & 0 & +1 \end{array}$

ي) $y = x^3 - x^2 + 2 \rightarrow y' = 3x^2 - 2x$
 $x(3x - 2) = 0 \rightarrow x = 0, x = \frac{2}{3}$

Sign chart: $\begin{array}{c|c|c|c|} - & 0 & + & + \\ \hline & -1/3 & 0 & 2/3 \end{array}$