

۱۹,۵ کامیاب و سرگرمی دراز و قدیم نمبر ۱۳

$$1) \Rightarrow \frac{f(x) + 0}{x^2 + 1} = \frac{9}{3} = 3 \Rightarrow \lim_{x \rightarrow 1^+} \frac{x^2 + 0}{x^2 + 1} = (3) \checkmark$$

$$2) \Rightarrow \frac{f(x) + 0}{x^2 + 1} = \frac{9}{3} = 3 \Rightarrow \lim_{x \rightarrow 1^-} \frac{x^2 + 0}{x^2 + 1} = (3) \checkmark$$

$$3) \Rightarrow \frac{f(x) + 0}{x^2 + 1} = \frac{9}{3} = 3 \checkmark$$

$$4) \Rightarrow \text{اولی برکت} \Rightarrow \left[\frac{x^2}{x^2 + 1} \right]_{0^+} = 0 \Rightarrow \frac{f(0) + 0}{0} = \frac{0}{0} = \text{نمی‌تواند} \checkmark$$

$$\lim_{x \rightarrow 1^+} f(x) - 2 \Rightarrow \text{اولی برکت} \Rightarrow f(x) - 2 = (6) \checkmark$$

$$1) \lim_{x \rightarrow 1^-} [f(x) - 2] \Rightarrow f(x) = 13 \Rightarrow [13 - 2] = (9) \checkmark$$

$$\Rightarrow \lim_{x \rightarrow 1^-} [f(x) - 2] = 9$$

$$2) \left[\lim_{x \rightarrow 1^-} f(x) - 2 \right] = (10) \checkmark$$

$$3) \left[\lim_{x \rightarrow 1^+} f(x) - 2 \right] = (10) \checkmark$$

$$1) \lim_{x \rightarrow 1^+} \frac{f(x) + 1}{x} \Rightarrow \frac{f(x) + 1}{x} = \left(\frac{1}{1} \right) \checkmark$$

$$2) \lim_{x \rightarrow 1^-} \left[\frac{f(x) + 1}{x} \right] \Rightarrow \left[\frac{f(x) + 1}{x} \right] = (5) \checkmark$$

$$3) \left[\lim_{x \rightarrow 1^+} \frac{f(x) + 1}{x} \right] \Rightarrow \left[\frac{f(x) + 1}{x} \right] \Rightarrow (5) \checkmark$$

$$4) \left[\lim_{x \rightarrow 1^+} \frac{f(x) + 1}{x} \right] = (9) \checkmark$$

$$الف) \lim_{x \rightarrow 1} \frac{x-5}{x^2-4x+5} \Rightarrow \frac{(x-5)}{(x-5)(x-1)} = \frac{1}{(x-1)} \Rightarrow \lim_{x \rightarrow 1} \frac{1}{(x-1)}$$

$$\left\{ \begin{array}{l} x^+ \\ x^- \end{array} \right\} \frac{1}{(x-1)} = \frac{1}{0^+} = +\infty$$

$$\frac{1}{(x-1)} = \frac{1}{0^-} = -\infty$$

$$ب) \lim_{x \rightarrow 2} \frac{(x-4)}{x^2-4x+9} \Rightarrow \left\{ \begin{array}{l} x^+ \\ x^- \end{array} \right\} \frac{(x^2-4)}{(x^2-11x+9)} = \frac{-1^+}{0^+} = -\infty$$

$$\frac{(x^2-4)}{(x^2-11x+9)} = \frac{-1^+}{0^+} = -\infty$$

$$الف) \lim_{x \rightarrow -3} [-x^2] - [5x]$$

$$\left\{ \begin{array}{l} x^+ \\ x^- \end{array} \right\} [-x^2] - [5x] = 0 + 15 = 15$$

$$[-x^2] - [5x] = 9 + 15 = 24$$

$$ب) \lim_{x \rightarrow (-\frac{1}{2})^-} \left[\frac{-1}{x^2} \right] = \left[-16 \right] = -16$$

به حال این است

$$الف) \left\{ \begin{array}{l} x^+ \\ x^- \end{array} \right\} \frac{x^2-1}{x^2-3} = \frac{(x-1)(x+1)}{(x-1)(x+3)} = \frac{x+1}{x+3} = \frac{1}{2}$$

$$\frac{-x^2+1}{(x^2-3)} = \frac{-(x-1)(x+1)}{(x-1)(x+3)} = \frac{-x-1}{(x+3)} = -\frac{1}{2}$$

$$ب) \left\{ \begin{array}{l} x^+ \\ x^- \end{array} \right\} \frac{-\sin x}{x \sin x \cos x} = \frac{-1}{x \cos x} = \frac{-1}{x \cos x} = \frac{1}{x}$$

$$\frac{\sin x}{x \sin x \cos x} = \frac{1}{x \cos x} = \frac{1}{x \cos x} = \frac{1}{x}$$

$$الف) \lim_{x \rightarrow \frac{\pi}{2}} \sqrt{\cos x}$$

$$\Rightarrow \sqrt{\cos \frac{\pi}{2}} = \sqrt{0} = 0$$

$$ب) \lim_{x \rightarrow 0} \sqrt{2-2\sqrt{x}}$$

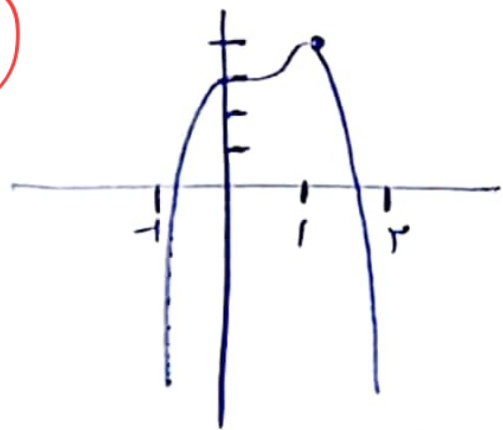
$$\Rightarrow 2-2\sqrt{x} \geq 0$$

$$2 \geq 2\sqrt{x} \Rightarrow x \in [0, 1]$$

در بازه $[0, 1]$ تابع $\sqrt{2-2\sqrt{x}}$ به ازای $x=0$ مقدار 1 را می‌گیرد و به ازای $x=1$ مقدار 0 را می‌گیرد.

$$f(x) = 4x^3 - 3x^4 + 3 \quad \text{عقب}$$

①



$$\text{الف) } \lim_{x \rightarrow 1} [4x^3 - 3x^4 + 3] = 4$$

$$\text{ب) } \lim_{x \rightarrow 0} [4x^3 - 3x^4 + 3] = 3$$

سوال ۹

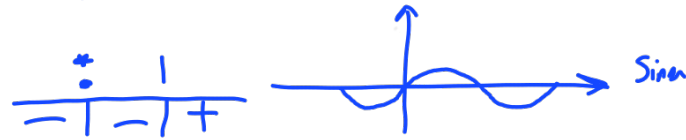
از روی شکل
رسم شده تابع

را در این مسئله

$$6) \lim_{x \rightarrow 0^+} \frac{(-1)^{\lfloor \sin x \rfloor}}{x^x - x^x} \Rightarrow \frac{0}{0} \Rightarrow \frac{(-1)^{\lfloor \sin x \rfloor}}{x^x - x^x} = \frac{(-1)^{\lfloor \sin x \rfloor}}{x^x(x^x - 1)} \Rightarrow \frac{1}{x^x(x^x - 1)} \Rightarrow \frac{1}{0^+} = +\infty \quad (9)$$

$$\lim_{x \rightarrow 0^+} \frac{(-1)^{\lfloor \sin x \rfloor}}{x^x - x^x} = \lim_{x \rightarrow 0^+} \frac{(-1)^{\lfloor \sin x \rfloor}}{x^x(x^x - 1)} = \lim_{x \rightarrow 0^+} \frac{1}{0^-} = -\infty$$

$$7) \lim_{x \rightarrow 2} \frac{x^x - \lfloor x^x \rfloor}{x - \lfloor x \rfloor}$$



(10)

$$\begin{aligned} & \text{for } x^+ \quad \lfloor x^+ \rfloor = \epsilon \Rightarrow \frac{x^+ - \epsilon}{x - 2} = \frac{(2-x)(2+x)}{x-2} = 2+x \\ & \text{for } x^- \quad \lfloor x^- \rfloor = 1 \Rightarrow \frac{x^- - 1}{x - 1} \Rightarrow \lim_{x \rightarrow 2^-} \frac{\epsilon - 1}{x - 1} = 1 \end{aligned}$$

$$\lim_{x \rightarrow 2} 2+x = \epsilon$$

(10)

$$0x^x - 1x^x = 10x^x - 10x^x \Rightarrow 10x^x - 10x^x \Rightarrow 10 - 10x^x$$

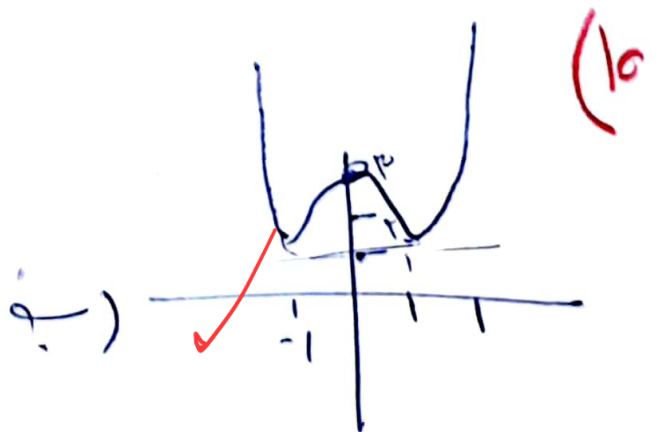


$$x^x - 1x^x = \epsilon x^x - \epsilon x^x \Rightarrow 1x^x - \epsilon$$



(10)

الن)



سؤال ٥

$$\text{الف) } \lim_{x \rightarrow -3} [-x] - [2x] = \begin{cases} \lim_{x \rightarrow (-3)^+} [-x] - [2x] = 1 - (-12) = 13 \\ \lim_{x \rightarrow (-3)^-} [-x] - [2x] = 4 - (-12) = 16 \end{cases}$$

$$\text{ب) } \lim_{x \rightarrow (-\frac{1}{4})^-} \left[\frac{-1}{x^4} \right] = -14$$

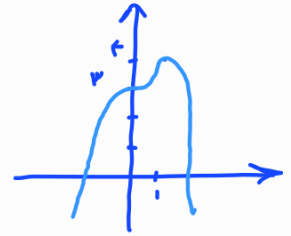
$$x < -\frac{1}{4} \rightarrow x^4 > \frac{1}{16} \rightarrow -x^4 < -\frac{1}{16} \rightarrow -\frac{1}{x^4} > -16$$

$$\text{الف) } \lim_{x \rightarrow 1} [x^3 - x^2 + 3] \rightarrow \lim_{x \rightarrow 1} [x] = 1$$

$$y = x^3 - x^2 + 3 \rightarrow y' = 3x^2 - 2x \xrightarrow{y'=0} 3x^2 - 2x = 0 \rightarrow \begin{cases} x=0 \\ x=1 \end{cases}$$

x	$-\infty$	0	1	$+\infty$
y'	+	+	-	
y		↗	↘	

سؤال ٤



$$\text{ب) } \lim_{x \rightarrow 0} [x^3 - x^2 + 3] = \begin{cases} \lim_{x \rightarrow 0^+} [x^3 - x^2 + 3] = 3 \\ \lim_{x \rightarrow 0^-} [x^3 - x^2 + 3] = 3 \end{cases}$$