

$$f(x) = x \left[\frac{x-2}{x} \right] + a \left[\frac{x+2}{x} \right] \quad \lim_{x \rightarrow -2^+} x \left[\frac{x-2}{x} \right] + a \left[\frac{x+2}{x} \right] = x \left[\frac{x-(x^+)}{x} \right] + a \left[\frac{-x^++2}{x} \right]$$

$$\lim_{x \rightarrow -2^-} x \left[\frac{x-2}{x} \right] + a \left[\frac{x+2}{x} \right] = x \left[\frac{x-(x^-)}{x} \right] + a \left[\frac{-x^-+2}{x} \right] = x(1) + a(0) = x$$

$$= x(2) + a(-1) = 2 - a$$

از آنجا که $x = -2$ در حد $\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^-} f(x) \rightarrow x = 2 - a \rightarrow a = 2 \rightarrow \left[\frac{a}{x} \right] = \left[\frac{2}{x} \right] = 0$

$$\lim_{x \rightarrow (\frac{\pi}{4})^-} [x \sin x - 1] \xrightarrow{\text{وقتی } x \rightarrow (\frac{\pi}{4})^- \text{ مقدار } \sin x \text{ کمتر است}} \lim_{x \rightarrow (\frac{\pi}{4})^-} [x \sin x - 1] = \left[x \left(\frac{1}{x} \right) - 1 \right] = [1 - 1] = [0]$$

$$= -1$$

$$\lim_{x \rightarrow -\frac{1}{x}} [1x^3 - x] = \left[1 \left(\frac{1}{x} \right)^3 - \left(\frac{1}{x} \right) \right] = \left[-1 + \frac{1}{x} \right] = \left[-\frac{1}{x} \right] = -1$$

چون $x = \frac{1}{x}$ نیست پس نیازی به محاسبه حد چپ و راست نبود

$$\lim_{x \rightarrow (\frac{1}{x})^-} \frac{\log x - 5 + \left[\frac{x}{x^2} \right]}{14x - \left[-\frac{1}{x^2} \right]} \xrightarrow{(1)(2)} \lim_{x \rightarrow (\frac{1}{x})^-} \frac{\log x - 5 + 1}{14x - (-1)} = \lim_{x \rightarrow (\frac{1}{x})^-} \frac{\log x + 4}{14x + 1} = \frac{1}{0^-} = -\infty$$

محد ندارد

$$x < -\frac{1}{x} \rightarrow x > \frac{1}{x} \rightarrow \frac{1}{x^2} < 1 \rightarrow \begin{cases} \frac{x}{x^2} < 1 \rightarrow \left[\frac{x}{x^2} \right] = 0 & (I) \\ \frac{x}{x^2} > 1 \rightarrow \left[\frac{x}{x^2} \right] = 1 & (II) \end{cases}$$

$$\lim_{x \rightarrow 1^+} \frac{\sin^2 \pi x}{[x] + \cos \pi x} \stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 1^+} \frac{1 - \cos^2 \pi x}{1 + \cos \pi x} = \lim_{x \rightarrow 1^+} \frac{(1 - \cos \pi x)(1 + \cos \pi x)}{(1 + \cos \pi x)}$$

$$= \lim_{x \rightarrow 1^+} (1 - \cos \pi x) = 1 - \cos \pi = 1 - (-1) = 2$$

$$\lim_{x \rightarrow -1} \frac{\sqrt{x^2+3} - \sqrt{x^2+4}}{1+\sqrt{x}} = \frac{0}{0} \text{ مبهم}$$

$$\rightarrow \lim_{x \rightarrow -1} \frac{\sqrt{x^2+3} - \sqrt{x^2+4}}{1+\sqrt{x}} \times \frac{(1-\sqrt{x}+\sqrt{x^2})}{(1-\sqrt{x}+\sqrt{x^2})} \times \frac{(\sqrt{x^2+3}+\sqrt{x^2+4})}{(\sqrt{x^2+3}+\sqrt{x^2+4})}$$

$$= \lim_{x \rightarrow -1} \frac{x^2+3-x^2-4}{1+x} \times \frac{1-\sqrt{x}+\sqrt{x^2}}{\sqrt{x^2+3}+\sqrt{x^2+4}} = (-1) \times \frac{3}{2} = -\frac{3}{2}$$

$$\lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x^2+1}}{x^2 - \sqrt{x^2+1}} = \frac{0}{0} \text{ مبهم} \rightarrow \lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x^2+1}}{x^2 - \sqrt{x^2+1}} \times \frac{x^2 + \sqrt{x^2+1}}{x^2 + \sqrt{x^2+1}}$$

$$= \lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x^2+1}}{x^2 - \sqrt{x^2+1}} \times (x^2 + \sqrt{x^2+1}) = \lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x^2+1}}{(x-1)(x+1)} \times (x^2 + \sqrt{x^2+1})$$

$$= \frac{x^2 - \sqrt{x^2+1}}{(x-1)(x+1)} \times (x^2 + \sqrt{x^2+1}) = -\frac{4}{2} = -2$$

$$\lim_{x \rightarrow 0} \frac{a + \sqrt{bx+c}}{x} = \frac{1}{4}$$

چون دارای حد است درون حد خارج کسر وقتی $x \rightarrow 0$ ، پس صورت کسر نیز وقتی $x \rightarrow 0$ باید برابر صفر شود

$$\rightarrow \lim_{x \rightarrow 0} (a + \sqrt{bx+c}) = 0 \rightarrow a + \sqrt{c} = 0 \rightarrow a = -\sqrt{c}$$

$$\lim_{x \rightarrow 0} \frac{a + \sqrt{bx+c}}{x} = \frac{1}{4} \xrightarrow{\text{HOP}} \lim_{x \rightarrow 0} \frac{\frac{b}{2\sqrt{bx+c}}}{1} = \frac{1}{4} \rightarrow \frac{b}{2\sqrt{c}} = \frac{1}{4} \rightarrow b = \frac{\sqrt{c}}{2} \rightarrow \frac{ab}{c} = \frac{-\sqrt{c} \times \frac{\sqrt{c}}{2}}{c} = -\frac{1}{2}$$

$$\lim_{x \rightarrow -\pi} \frac{x + k[\frac{x}{\pi}]}{\sin x} = +\infty \quad \lim_{x \rightarrow -\pi} \frac{x + k[\frac{x}{\pi}]}{\sin x} = +\infty \rightarrow \frac{x + k[\frac{x}{\pi}]}{0^-} = +\infty$$

$$\rightarrow x + k[\frac{x}{\pi}] < 0 \rightarrow k[\frac{x}{\pi}] < -x \rightarrow k[\frac{-\pi}{\pi}] < -(-\pi) \rightarrow -k < \pi \rightarrow k > -\pi$$

$$\lim_{x \rightarrow -\pi} \frac{x + k[\frac{x}{\pi}]}{\sin x} = +\infty \rightarrow \frac{x + k[\frac{x}{\pi}]}{0^+} = +\infty \rightarrow x + k[\frac{x}{\pi}] > 0 \rightarrow k[\frac{x}{\pi}] > -x$$

$$\rightarrow k[\frac{-\pi}{\pi}] > -(-\pi) \rightarrow -k > \pi \rightarrow k < -\pi$$

$$\lim_{x \rightarrow \infty} \frac{b\sqrt{x^2+1} - 2b}{ax-b} = \frac{0}{0} \xrightarrow{\text{HOP}} \lim_{x \rightarrow \infty} \frac{b}{\frac{x^2+1}{x} - 2} = \frac{b}{\frac{1}{x} + 2}$$

$$\text{صورت کسر برابر صفر است: } \lambda a - b = 0 \rightarrow b = \lambda a$$

$$\xrightarrow{b = \lambda a} \frac{\lambda a}{\frac{1}{x} + 2} = \frac{1}{4}$$