پارسا انوری - تکلیف ۱۹

-۱

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} f(x)$$

$$\lim_{x\to -2^+} 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right] = 2\times[2^-] + a[0^+] = 2$$

$$\lim_{x\to -2^-} 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right] = 2\times[2^+] + a[0^-] = 4-a$$

$$2 = 4 - a \longrightarrow a = 2$$

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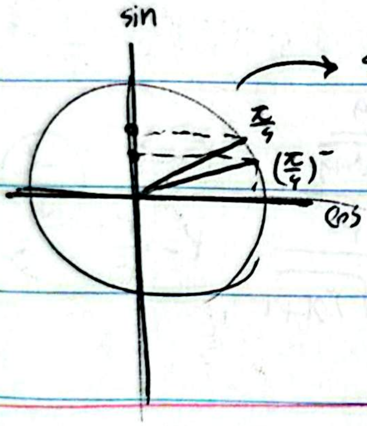

$$\sin\left(\frac{\pi}{6}\right)^- < \frac{1}{2}$$

$$\lim_{x\to (\frac{\pi}{6})^-} [2\sin x - 1] = [1^- - 1] = [0^-] = -1$$

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$$\lim_{x\to -\frac{1}{2}} [8x^3 - x] = \left[8\times\frac{-1}{8} + \frac{1}{2}\right] = \left[\frac{-1}{2}\right] = -1$$

-۴

$$\lim_{x\to (\frac{-1}{2})^-} \frac{10x - 5 + \left[\frac{3}{x^2}\right]}{16x - \left[\frac{-2}{x^2}\right]} = \frac{-5^- - 5 + [12^-]}{-8^- - [-8^+]} = \frac{-10^- + 11}{-8^- + 8} = \frac{1^-}{0^-} = -\infty$$

$$x < \frac{-1}{2} \to x^2 > \frac{1}{4} \to \frac{1}{x^2} < 4 \longrightarrow \frac{3}{x^2} < 12$$

$$\longrightarrow \frac{-2}{x^2} > -8$$

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$$\lim_{x\to 1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\frac{\sin^2\pi}{[1^+]+\cos\pi}=\frac{\sin^2\pi}{1+\cos\pi}=\frac{(1+\cos\pi)(1-\cos\pi)}{1+\cos\pi}=$$

$$1-\cos\pi=2$$

۶-

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}\xRightarrow{\text{هوپیتال}}\frac{\frac{2}{2\sqrt{2x+3}}-\frac{3}{2\sqrt{3x+4}}}{\frac{1}{3\sqrt[3]{x^2}}}=\frac{1-\frac{3}{2}}{\frac{1}{3}}=\frac{-3}{2}$$

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$$\sqrt{49x}$$

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}\xRightarrow{\text{هوپیتال}}\frac{2-\frac{49}{2\sqrt{49x}}}{2-\frac{3}{2\sqrt{3x+1}}}=\frac{2-\frac{7}{2}}{2-\frac{3}{4}}=\frac{-6}{5}$$

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$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}\to\frac{a+\sqrt{c}}{0}=\frac{1}{4}\to a+\sqrt{c}=0$$

$$a=-\sqrt{c}$$

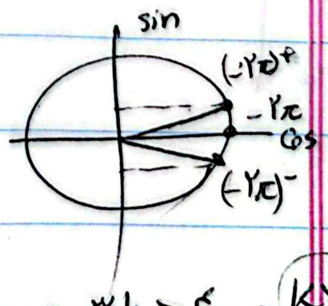
چون $\frac{0}{0}\xRightarrow{\text{هوپیتال}}\lim_{x\to 0}\frac{b}{\cancel{2}\sqrt{bx+c}}=\frac{1}{\cancel{4}\,2}\to 2b=\sqrt{c}\to b=\frac{\sqrt{c}}{2}$

$$\frac{ab}{c}=\frac{-\sqrt{c}\times\sqrt{c}}{2c}=\frac{-1}{2}$$

۹-

$$\lim_{x\to -2\pi^+}\frac{4+k[\frac{x}{\pi}]}{\sin x}=\frac{4-2k}{0^+}=+\infty\to 4-2k>0\to k<2 \quad (\mathrm{I})$$

$$\lim_{x\to -2\pi^-}\frac{4+k[\frac{x}{\pi}]}{\sin x}=\frac{4-3k}{0^-}=+\infty$$



$$4-3k<0\to 3k>4\to k>\frac{4}{3} \quad (\mathrm{II})$$

$$(\mathrm{I}),(\mathrm{II})\longrightarrow\frac{4}{3}<k<2\longrightarrow -2<-k<\frac{-4}{3}$$

$$[-k]=-2$$

$$\lim_{x \to 8} \frac{b\sqrt{2+\sqrt[3]{x}} - 2b}{ax - b} = \frac{2b - 2b}{8a - b} = \frac{0}{8a - b} \xrightarrow[\text{غیرصفر است}]{\text{چون جواب}} 8a = b$$

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$$\lim_{x \to 8} \frac{8a\sqrt{2+\sqrt[3]{x}} - 2b}{ax - b} \xrightarrow{\text{هوپیتال}} \frac{8\cancel{a} \times \dfrac{\dfrac{1}{3\sqrt[3]{x^2}}}{2\sqrt{2+\sqrt[3]{x}}}}{\cancel{a}} = \frac{8 \times \frac{1}{12}}{4} = \frac{1}{6}$$