مبین شهریاری ۱۶٫۵ -۱

$$f(x)=2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]$$

$$\lim_{x\to -2}2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]$$

$$\lim_{x\to -2^+}2\left[\cancel{\frac{2-x}{2}}\right]^{1}+a\left[\cancel{\frac{x+2}{3}}\right]^{0}=2+0=2$$

$$\lim_{x\to -2^-}2\left[\cancel{\frac{2-x}{2}}\right]^{2}+a\left[\cancel{\frac{x+2}{3}}\right]^{-1}=4-a$$

$$4-a=2\Rightarrow \boxed{a=2}\qquad \left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=0$$

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-۲

$$\lim_{x\to \frac{\pi}{6}^-}[2\sin x-1]=[2\cancel{\sin\tfrac{\pi}{6}}^{\,1^-}_{\,\frac{1}{2}^-}-1]=[0^-]=-1$$

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-۳

$$\lim_{x\to -\frac{1}{2}}[8x^3-x]$$

$$\lim_{x\to -\frac{1}{2}^+}[8x^3-x]=[\cancel{8x^3}]^{-1}+[\cancel{-x}]^{0}=-1 \qquad -x<\frac{1}{2}\quad x>-\frac{1}{2}\quad x^3>-\frac{1}{8}\quad 8x^3>-1$$

$$\lim_{x\to -\frac{1}{2}^-}[8x^3-x]\longrightarrow[-1^-+\tfrac{1}{2}^+]=-1$$

$$x<-\frac{1}{2}\quad x^3<-\frac{1}{8}\quad 8x^3<-1\quad -x>\frac{1}{2}$$

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$$x\to\left(-\tfrac{1}{2}\right)^-\longrightarrow x<-\frac{1}{2}\to x^2>\frac{1}{4}\to\frac{1}{x^2}<4\to\frac{3}{x^2}<12\to\left[\frac{3}{x^2}\right]=11$$

$$\frac{1}{x^2}<4\to-\frac{1}{x^2}>-4\to\frac{-2}{x^2}>-8\to\left[\frac{-2}{x^2}\right]=-8$$

$$\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+\left[\cancel{\frac{3}{x^2}}\right]^{[12^+]}}{16x-\left[\cancel{-\frac{2}{x^2}}\right]_{[-8^-]}}=\lim\frac{10x-5+12}{16x-(-9)}=\frac{10x+7}{16x+9}=\frac{-5+7}{-8+9}=2$$

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$$\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+11}{16x-(-8)}=\lim_{x\to(-\frac{1}{2})^-}\frac{10x+6}{16x+8}=\frac{-5+6}{(-8)^-+8}=\frac{1}{0^-}=-\infty$$

$$x<-\frac{1}{2}\to x^2<\frac{1}{4}\to\frac{1}{x^2}>4\qquad\frac{3}{x^2}>12$$

$$\frac{1}{x^2}>4\qquad-\frac{1}{x^2}\cancel{>}-4\qquad\frac{-2}{x^2}<-8$$

-۵

~~با استفاده از اتحاد~~ $\cancel{\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}}$

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$$\lim_{x\to 1^+} \frac{\sin^2 \pi x}{[x]+\cos \pi x} = \frac{\sin^2 \pi x}{1+\cos \pi x} = \cancel{\frac{\sin^2 \pi x}{\sin \pi x}} = \cancel{\sin \pi x \Rightarrow \sin \pi = 0}$$

$$\to \frac{1-\cos^2 \pi x}{1+\cos \pi x} = \frac{\cancel{(1+\cos\pi x)}(1-\cos \pi x)}{\cancel{1+\cos \pi x}} = 1-\cos\pi = 1-(-1) = 2$$

-۶

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$$\lim_{x\to -1} \frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \Rightarrow \frac{0}{0} \xrightarrow{\text{رفع ابهام}} \frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \times \frac{\sqrt{2x+3}+\sqrt{3x+4}}{\sqrt{2x+3}+\sqrt{3x+4}}$$

$$\frac{2x+3-3x-4}{(1+\sqrt[3]{x})(\sqrt{2x+3}+\sqrt{3x+4})} \times \frac{1-\sqrt[3]{x}+\sqrt[3]{x^2}}{1-\sqrt[3]{x}+\sqrt[3]{x^2}} = \frac{-\cancel{(x+1)}(1-\sqrt[3]{x}+\sqrt[3]{x^2})}{\cancel{(1+x)}(\sqrt{2x+3}+\sqrt{3x+4})} = \frac{-(1-(-1)+1)}{2} = -\frac{3}{2}$$

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۱,۵

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} = \frac{0}{0} \xrightarrow[\text{ابهام}]{\text{رفع}} \frac{(\sqrt{x}-\frac{5}{2})(\sqrt{x}-1)}{2x-\sqrt{3x+1}} \times \frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}} = \frac{(\sqrt{x}-\frac{5}{2})(\sqrt{x}-1)(2x+\sqrt{3x+1})}{4x^2-3x-1}$$

$$\frac{(\sqrt{x}-\frac{5}{2})\cancel{(\sqrt{x}-1)}(2x+\sqrt{3x+1})}{\cancel{(x-1)}(x+\frac{1}{4})(\sqrt{x}+1)} = \frac{(1-\frac{5}{2})\cancel{(4)}}{\cancel{(2)}(\frac{5}{4})} = \frac{-6}{\cancel{\frac{5}{2}}} = -2{,}4 \longrightarrow \frac{-12}{10}$$

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)}{2x-\sqrt{3x+1}} = \frac{(\sqrt{x}-1)(2\sqrt{x}-5)}{2x-\sqrt{3x+1}} \times \frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}} \longrightarrow$$

$$\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)\times(2x+\sqrt{3x+1})}{4x^2-3x-1} = \lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)(4)}{(4x+1)(x-1)} =$$

$$(x-1) \rightarrow (\sqrt{x}-1)(\sqrt{x}+1)$$

$$\lim_{x\to 1}\frac{(2\sqrt{x}-5)(4)}{(4x+1)(\sqrt{x}+1)} = \frac{(-3)(4)}{5\times 2} = -1{,}2$$

۸-

$$\lim_{x\to ۰}\frac{a+\sqrt{bx+c}}{x}=\frac{۱}{۴}\qquad \frac{\frac{b}{۲\sqrt{bx+c}}}{۱}=\frac{۱}{۴}$$

$b=۱$

$c=۴$

ابتدا صفر صفر بود $\rightarrow a=-\sqrt{bx+c}$ $\quad \boxed{a=-۲}$

$$\frac{ab}{c}=\frac{-۲\times ۱}{۴}=-\frac{۱}{۲}$$

۴

$$\lim_{x\to -۲\pi}\frac{۴+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty \longrightarrow \frac{۴+k\left[\frac{x}{\pi}\right]}{۰^{\sim}}=+\infty$$

$$۴+k\left[\frac{x}{\pi}\right]>۰$$

$\sin ۲\pi=۰$

$$۴+k\left[\frac{-۲\pi}{\pi}\right]\qquad ۴+$$

۹-

$$\lim_{x\to -۲\pi}\frac{۴+k\left[\frac{x}{\pi}\right]}{\sin x}$$

$$\lim_{x\to -۲\pi^{+}}\frac{۴+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{۴-۲k}{۰}=+\infty\qquad ۴-۲k>۰\quad k<۲\quad -k>-۲$$

$$[-k]=-۲$$

$$\lim_{x\to -۲\pi^{-}}\frac{۴+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{۴-۳k}{۰}=+\infty\qquad ۴-۳k>۰\quad k<\frac{۴}{۳}\quad -k>-\frac{۴}{۳}$$

$$[-k]=-۲$$

۲

۱۰.

$$\lim_{x\to ۸}\frac{۸a\left(\sqrt{۲+\sqrt[۳]{x}}-۲\right)}{ax-۸a}=\frac{۸a\left(\sqrt{۲+\sqrt[۳]{x}}-۲\right)}{ax-۸a}\times\frac{\sqrt{۲+\sqrt[۳]{x}}+۲}{\sqrt{۲+\sqrt[۳]{x}}+۲}=$$

۱

$$\lim_{x\to ۸}\frac{b\sqrt{۲+\sqrt[۳]{x}}-۲b}{ax-b}=\frac{\cancel{b}\left(\sqrt{۲+\sqrt[۳]{x}}-۲\right)}{\cancel{b}\left(\frac{x}{۸}-۱\right)}\times\frac{\sqrt{۲+\sqrt[۳]{x}}+۲}{\underbrace{\cancel{\sqrt{۲+\sqrt[۳]{x}}+۲}}_{۴}}=\frac{۲+\sqrt[۳]{x}-۴}{\left(\frac{x}{۲}-۴\right)}$$

$۸a-b=۰$

$۸a=b$

$a=\frac{b}{۸}$

$$\frac{\sqrt[۳]{x}-۲}{\frac{x}{۲}-۴}\xrightarrow{hop}\frac{\frac{۱}{۳\sqrt[۳]{x^{۲}}}}{\frac{۱}{۲}}=\frac{۲}{۳\sqrt[۳]{x^{۲}}}=\frac{۲}{۴}=\frac{۱}{۲}$$

$$\lim_{x\to ۸}\frac{۸a\left(\sqrt[۳]{x}-۲\right)}{a(x-۸)\times ۴}\rightarrow\lim_{x\to ۸}\frac{۲\left(\sqrt[۳]{x}-۲\right)}{\left(\sqrt[۳]{x}-۲\right)\left(\sqrt[۳]{x^{۲}}+۴+۲\sqrt[۳]{x}\right)}=\frac{۲}{۱۲}=\frac{۱}{۶}$$