۱-

$$f(x) = ۲\left[\frac{۲-x}{۲}\right] + a\left[\frac{x+۲}{۳}\right]$$

$$\lim_{x\to -۲} ۲\left[\frac{۲-x}{۲}\right] + a\left[\frac{x+۲}{۳}\right]$$

$$\to \lim_{x\to -۲^+} ۲\left[\cancel{\frac{۲-x}{۲}}\right] + a\left[\cancel{\frac{x+۲}{۳}}\right] = ۲ + ۰ = ۲$$

$$\to \lim_{x\to -۲^-} ۲\left[\cancel{\frac{۲-x}{۲}}\right] + a\left[\cancel{\frac{x+۲}{۳}}\right] = ۴ - a$$

$$۴ - a = ۲ \Rightarrow \boxed{a = ۲}$$

$$\left[\frac{a}{۳}\right] = \left[\frac{۲}{۳}\right] = ۰$$

۲-

$$\lim_{x\to \frac{\pi}{۶}^-} [۲\sin x - ۱] = \left[۲\cancel{\sin\frac{\pi}{۶}^-}_{\frac{۱}{۲}} - ۱\right] = [۰^-] = -۱$$

۳-

$$\lim_{x\to -\frac{۱}{۲}} [۸x^۳ - x]$$

$$\to \lim_{x\to -\frac{۱}{۲}^+} [۸x^۳ - x] = [\cancel{۸x^۳}] + [\cancel{-x}] = -۱$$

$-x < \frac{۱}{۲}$ $\quad x > -\frac{۱}{۲}$ $\quad x^۳ > -\frac{۱}{۸}$ $\quad ۸x^۳ > -۱$

$$\to \lim_{x\to -\frac{۱}{۲}^-} [۸x^۳ - x] \to \left[-۱^- + \frac{۱}{۲}^+\right] = -۱$$

$x < -\frac{۱}{۲}$ $\quad x^۳ < -\frac{۱}{۸}$ $\quad ۸x^۳ < -۱$

$-x > \frac{۱}{۲}$

۴-

$$\lim_{x\to (-\frac{۱}{۲})^-} \frac{۱۰x - ۵ + \overset{[۱۲^+]}{\cancel{\left[\frac{۳}{x^۲}\right]}}}{۱۶x - \underset{[-۸^-]}{\cancel{\left[-\frac{۲}{x^۲}\right]}}} = \lim \frac{۱۰x - ۵ + ۱۲}{۱۶x - (-۹)} = \frac{۱۰x + ۷}{۱۶x + ۹} = \frac{-۵ + ۷}{-۸ + ۹} = ۲$$

$x < -\frac{۱}{۲} \to x^۲ < \frac{۱}{۴} \to \frac{۱}{x^۲} > ۴$ $\quad \frac{۳}{x^۲} > ۱۲$

$\frac{۱}{x^۲} > ۴$ $\quad -\frac{۱}{x^۲} \cancel{<} -۴$ $\quad \frac{-۲}{x^۲} < -۸$

۵-

$$\cancel{\frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}}$$

$$\lim_{x\to 1^+} \frac{\sin^2 \pi x}{[x] + \cos \pi x} = \frac{\sin^2 \pi x}{1+\cos \pi x} = \cancel{\frac{\sin^2 \pi x}{\sin \pi x} = \sin \pi x \Rightarrow \sin \pi = 0}$$

$$\rightarrow \frac{1-\cos^2 \pi x}{1+\cos \pi x} = \frac{\cancel{(1+\cos \pi x)}(1-\cos \pi x)}{\cancel{1+\cos \pi x}} = 1-\cos \pi = 1-(-1) = 2$$

۶-

$$\lim_{x\to -1} \frac{\sqrt{2x+3} - \sqrt{3x+4}}{1+\sqrt[3]{x}} \Rightarrow \frac{0}{0} \xrightarrow{\text{رفع ابهام}} \frac{\sqrt{2x+3} - \sqrt{3x+4}}{1+\sqrt[3]{x}} \times \frac{\sqrt{2x+3} + \sqrt{3x+4}}{\sqrt{2x+3} + \sqrt{3x+4}}$$

$$\frac{2x+3-3x-4}{(1+\sqrt[3]{x})(\sqrt{2x+3} + \sqrt{3x+4})} \times \frac{1-\sqrt[3]{x}+\sqrt[3]{x^2}}{1-\sqrt[3]{x}+\sqrt[3]{x^2}} = \frac{-\cancel{(x+1)}(1-\sqrt[3]{x}+\sqrt[3]{x^2})}{\cancel{(1+x)}(\sqrt{2x+3} + \sqrt{3x+4})} = \frac{-(1-(-1)+1)}{2} = -\frac{3}{2}$$

-۷

$$\lim_{x\to ۱} \frac{۲x-۷\sqrt{x}+۵}{۲x-\sqrt{۳x+۱}} = \frac{۰}{۰} \xrightarrow[\text{ابهام}]{\text{رفع}} \frac{\left(\sqrt{x}-\frac{۵}{۲}\right)\left(\sqrt{x}-۱\right)}{۲x-\sqrt{۳x+۱}} \times \frac{۲x+\sqrt{۳x+۱}}{۲x+\sqrt{۳x+۱}} = \frac{\left(\sqrt{x}-\frac{۵}{۲}\right)\left(\sqrt{x}-۱\right)\left(۲x+\sqrt{۳x+۱}\right)}{۴x^۲-۳x-۱}$$

$$\frac{\left(\sqrt{x}-\frac{۵}{۲}\right)\cancel{\left(\sqrt{x}-۱\right)}\left(۲x+\sqrt{۳x+۱}\right)}{\cancel{(x-۱)}\left(x+\frac{۱}{۴}\right)\left(\sqrt{x}+۱\right)} = \frac{\overset{-\frac{۳}{۲}}{\cancel{\left(۱-\frac{۵}{۲}\right)}}\left(۴\right)}{(۲)\left(\frac{۵}{\cancel{۴}\,۲}\right)} = \frac{-۶}{\frac{۵}{۲}} = -۲,۴$$

۸-

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4} \qquad \frac{\frac{b}{2\sqrt{bx+c}}}{1}=\frac{1}{4}$$

$$b=1$$

$$c=4$$

ابتدا صفر صفر بود $\rightarrow a=-\sqrt{bx+c}=\boxed{a=-2}$

$$\frac{ab}{c}=\frac{-2\times 1}{4}=-\frac{1}{2}$$

$$\lim_{x\to -2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty \longrightarrow \frac{4+k\left[\frac{x}{\pi}\right]}{0^{\sim}}=+\infty$$

$$4+k\left[\frac{x}{\pi}\right]>0$$

$$\sin 2\pi=0 \qquad 4+k\left[\frac{-2\pi}{\pi}\right] \qquad 4+$$

۹-

$$\lim_{x\to -2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}$$

$$\rightarrow \lim_{x\to -2\pi^+}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{4-2k}{0}=+\infty \quad 4-2k>0 \quad k<2 \quad -k>-2$$

$$[-k]=-2$$

$$\rightarrow \lim_{x\to -2\pi^-}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{4-3k}{0}=+\infty \quad 4-3k>0 \quad k<\frac{4}{3} \quad -k>-\frac{4}{3}$$

$$[-k]=-2$$

۱۰.

$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}=\frac{\cancel{b}\left(\sqrt{2+\sqrt[3]{x}}-2\right)}{\cancel{b}\left(\frac{x}{8}-1\right)}\times\frac{\sqrt{2+\sqrt[3]{x}}+2}{\underbrace{\sqrt{2+\sqrt[3]{x}}+2}_{4}}=\frac{2+\sqrt[3]{x}-4}{\left(\frac{x}{2}-4\right)}$$

$$8a-b=0$$
$$8a=b$$
$$a=\frac{b}{8}$$

$$\frac{\sqrt[3]{x}-2}{\frac{x}{2}-4}\xrightarrow{Hop}\frac{\frac{1}{\sqrt[3]{x^2}}}{1/2}=\frac{2}{\sqrt[3]{x^2}}=\frac{2}{4}=\frac{1}{2}$$