آراد آزادنیا

تکلیف شماره ۱۹

دوازدهم تجربی ۸

①

$$\lim_{x\to -2^+} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] = 2[2]+a\left[\frac{0^+}{3}\right] = 2$$

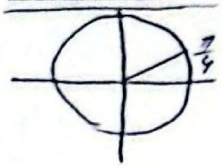
$$\Longrightarrow 2 = 4-a \Rightarrow a=2$$

$$\lim_{x\to -2^-} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] = 2[2^+]+a\left[\frac{0^-}{3}\right] = 4-a$$

$$\left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=0$$

②

$$\lim_{x\to(\frac{\pi}{6})^-}[2\sin x-1] \xrightarrow{\sin\frac{\pi}{6}=\frac{1}{2}} \left[2\left(\frac{1}{2}^-\right)-1\right]=[1^- -1]=-1$$



③

$$\lim_{x\to -\frac{1}{2}}[8x^3-x] = \left[8\times\frac{-1}{8}+\frac{1}{2}\right] = \left[-1+\frac{1}{2}\right] = \left[-\frac{1}{2}\right] = -1$$

طبیعتاً چون داخل براکت عدد صحیح نمی‌شود ۲ شاخه‌ای نمی‌کنیم.

④

$$\lim_{x\to -\frac{1}{2}^-}\frac{10x-5+\left[\frac{3}{x^2}\right]}{16x-\left[\frac{-2}{x^2}\right]} \xrightarrow{\text{تکلیف براکت را مشخص می‌کنیم}}$$

$$x<\frac{-1}{2} \Rightarrow \frac{1}{x}>-2 \Rightarrow \frac{1}{x^2}<4 \Rightarrow \frac{3}{x^2}<12 \quad [12^-]=11$$

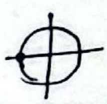
$$\frac{1}{x^2}<4 \Rightarrow \frac{-2}{x^2}>-8 \longrightarrow [-8^+]=-8$$

$$\lim_{x\to -\frac{1}{2}^-}\frac{10x-5+11}{16x+8} = \frac{10x+6}{16x+8} = \frac{-5+6}{-8^-+8} = \frac{1}{0^-} \xrightarrow[\text{صفر}]{\text{عکس}} -\infty$$

⑤

$$\lim_{x\to 1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x} \xrightarrow{\text{تکلیف براکت}} \frac{\sin^2\pi x}{1+\cos\pi x} = \frac{1-\cos^2\pi x}{1+\cos\pi x} = \frac{(1-\cos\pi x)\cancel{(1+\cos\pi x)}}{\cancel{1+\cos\pi x}} = 1-\cos\pi x$$

$$\lim_{x\to 1^+} 1-\cos\pi x = 1-\cos\pi^+ = 1-(-1) = 2$$



⑥

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \Longrightarrow \times\frac{\cancel{\text{ص ۲}}}{\cancel{\text{ص ۲}}}\times\frac{\cancel{\text{ف ۲}}}{\cancel{\text{ف ۲}}} = \frac{2x+3-3x-4}{x+1}\times\frac{\text{ف ۲}}{\text{ص ۲}}$$

$$=\lim_{x\to -1}\frac{-\cancel{(x+1)}\times \text{ف ۲}}{\cancel{x+1}\times \text{ص ۲}} = \frac{-3}{2}$$

(۷)

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \xrightarrow{HoP\ \frac{0}{0}} \lim_{x\to 1}\frac{2-\frac{7}{2\sqrt{x}}}{2-\frac{3}{2\sqrt{3x+1}}}=\frac{2-\frac{7}{2}}{2-\frac{3}{4}}=\frac{\frac{-3}{2}}{\frac{5}{4}}=\frac{-6}{5}$$

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(۸)

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4} \xrightarrow{①\ \frac{0}{0}\ \text{بودن}} a+\sqrt{c}=0 \Rightarrow a=-\sqrt{c}$$

$$② \lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x} \xrightarrow{HoP\ \frac{0}{0}} \frac{b}{2\sqrt{bx+c}}=\frac{1}{4} \Rightarrow \frac{b}{\sqrt{c}}=\frac{1}{2} \Rightarrow 2b=\sqrt{c} \Rightarrow b=\frac{\sqrt{c}}{2}$$

$$\Rightarrow \frac{ab}{c}=\frac{-\sqrt{c}\times\frac{\sqrt{c}}{2}}{c}=\frac{\frac{-c}{2}}{c}=\frac{-1}{2}$$

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(۹)

$$\lim_{x\to -2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty$$

$$\xrightarrow{-2\pi^+} \frac{4+k[-2^+]}{0^+}=+\infty \xrightarrow{\text{عدد مثبت}} 4-2k>0 \Rightarrow 2k<4,\quad k<2$$

$$\xrightarrow{-2\pi^-} \frac{4+k[-2^-]}{0^-}=+\infty \xrightarrow{\text{عدد مثبت}} 4-3k>0 \Rightarrow 3k<4 \Rightarrow k>\frac{4}{3}$$

$$\frac{4}{3}<k<2 \Rightarrow \frac{-4}{3}>-k>-2 \Rightarrow [-k]=-2$$

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(۱۰)

$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}=\text{عدد غیرصفر} \xrightarrow[\text{صورت صفر}]{\frac{0}{0}\ \text{بودن}} 8a-b=0 \Rightarrow b=8a \Rightarrow a=\frac{1}{8}b$$

$$b\left(\sqrt{2+\sqrt[3]{x}}-2\right)$$

$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b} \xrightarrow{HoP\ \frac{0}{0}} \frac{\cancel{b}\times\frac{1}{3}x^{\frac{-2}{3}}\times\frac{1}{2\sqrt{2+\sqrt[3]{x}}}}{\frac{1}{8}\cancel{b}}=8\left(\frac{1}{3}x^{\frac{-2}{3}}\right)\left(\frac{1}{2\sqrt{2+\sqrt[3]{x}}}\right)=\cancel{8}\times\frac{1}{\cancel{12}}\times\frac{1}{4}=\frac{1}{6}$$