

نام و نام خانوادگی: (بنا) پاسخنامه تشریحی تکلیف شماره ۹۰۰... کلاس: A

$$f(x) = x \left[\frac{x-1}{x} \right] + a \left[\frac{x+1}{x} \right] \quad (1) = (2) \rightarrow x = f - a \rightarrow a = x \rightarrow \left[\frac{x}{x} \right] = 1$$

$x \rightarrow -1$

$-1^+ \rightarrow x > -1$ $\begin{cases} x^- \rightarrow -x < 1 \xrightarrow{+1} 1-x < 2 \xrightarrow{\div x} \frac{1-x}{x} < 2 \rightarrow \left[\frac{1-x}{x} \right] = 1 \\ x^+ \rightarrow x+1 > 0 \xrightarrow{\div x} \frac{x+1}{x} > 0 \rightarrow \left[\frac{x+1}{x} \right] = 0 \end{cases} \rightarrow f(-1^+) = 1$

$-1^- \rightarrow x < -1$ $\begin{cases} x^- \rightarrow -x > 1 \xrightarrow{+1} 1-x > 2 \xrightarrow{\div x} \frac{1-x}{x} > 2 \rightarrow \left[\frac{1-x}{x} \right] = 2 \\ x^+ \rightarrow x+1 < 0 \xrightarrow{\div x} \frac{x+1}{x} < 0 \rightarrow \left[\frac{x+1}{x} \right] = -1 \end{cases} \rightarrow f(-1^-) = 2 - a = 1$

$$\lim_{x \rightarrow \frac{\pi}{4}^-} [x \sin x - 1]$$

$x < \frac{\pi}{4} \rightarrow \sin x < \frac{1}{\sqrt{2}} \xrightarrow{x^2} x \sin x < 1 \xrightarrow{-1} x \sin x - 1 < 0 \rightarrow [x \sin x - 1] = -1$

$$\lim_{x \rightarrow -\frac{1}{4}} [x^2 - x]$$

$x < -\frac{1}{4} \xrightarrow{x = -\frac{1}{4}} -\frac{1}{4}(2-1) = -\frac{1}{4} \rightarrow$ چون حاصل کسری منفی است نیازی به تطبیق در صورت و مخرج برای برابری نیست $\rightarrow \left[-\frac{1}{4} \right] = -1$

$$\lim_{x \rightarrow \left(-\frac{1}{4}\right)^-} \frac{10x - a + \left[\frac{x}{x^2}\right]}{14x - \left[-\frac{x}{x^2}\right]} = \frac{10x - \frac{1}{4} - a + 11}{14x - \frac{1}{4} - (-1)} = \frac{1}{0^-} = -\infty$$

$x < -\frac{1}{4} \xrightarrow{x^2 > \frac{1}{4}} \frac{1}{x^2} < 4 \xrightarrow{x^2} \frac{x}{x^2} < 4 \rightarrow \left[\frac{x}{x^2} \right] = 1$

$x < -\frac{1}{4} \xrightarrow{x^2 > \frac{1}{4}} \frac{1}{x^2} < 4 \xrightarrow{x-2} \frac{-2}{x^2} > -1 \rightarrow \left[\frac{-2}{x^2} \right] = -1$

$$\lim_{x \rightarrow 1^+} \frac{\sin^2 x}{[x] + \cos \pi x} = \frac{0}{1-1} = \frac{0}{0} \xrightarrow{\text{ل‌ه‌سپیتال}} \frac{1 - \cos^2 x}{1 + \cos \pi x} = \frac{(1 - \cos \pi x)(1 + \cos \pi x)}{1 + \cos \pi x} = 1 - \cos \pi x = 1 - (-1) = 2$$

$x > 1 \rightarrow [x] = 1$

$$\lim_{x \rightarrow -1} \frac{\sqrt{2x+3} - \sqrt{2x+1}}{1 + \sqrt{x}} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \frac{\sqrt{2x+3} - \sqrt{2x+1}}{1 + \sqrt{x}} \times \frac{\sqrt{2x+3} + \sqrt{2x+1}}{\sqrt{2x+3} + \sqrt{2x+1}} \times \frac{(1 + \sqrt{x}) - \sqrt{x}}{(1 + \sqrt{x}) - \sqrt{x}}$$

$$= \frac{(\sqrt{2x+3} - \sqrt{2x+1}) \times (1 + \sqrt{x}) - \sqrt{x}}{(\sqrt{2x+3} + \sqrt{2x+1}) \times (1 + x)} = \frac{-1 \times (1 + 1 - (-1))}{(1 + 1)} = \frac{-1}{2}$$

$$\lim_{x \rightarrow 1} \frac{2x - \sqrt{2x} + 1}{2x - \sqrt{2x} + 1} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \frac{2 - \frac{1}{\sqrt{2x}}}{2 - \frac{1}{\sqrt{2x}}} = \frac{2 - \frac{1}{\sqrt{2}}}{2 - \frac{1}{\sqrt{2}}} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$$

$$\lim_{x \rightarrow 0} \frac{a + \sqrt{bx+c}}{x} = \frac{1}{c} \quad , \quad \frac{ab}{c} = ? \rightarrow \frac{-\sqrt{c} \times \frac{\sqrt{c}}{c}}{c} = -\frac{1}{c}$$

چون مخرج صفر برداری نیست کسر 0 بوده و رافع را هم نزنه و همین حاصل 1/4

$$a + \sqrt{bx+c} \xrightarrow{x=0} a + \sqrt{c} = 0$$

$$\Rightarrow \sqrt{c} = -a \quad , \quad \sqrt{c} = \sqrt{b}$$

$$\text{hop} \rightarrow \frac{b}{\sqrt{bx+c}} \xrightarrow{x=0} \frac{b}{\sqrt{c}} = \frac{1}{c} \rightarrow \sqrt{c} = \sqrt{b} \quad \downarrow \quad \downarrow$$

$$a = -\sqrt{c} \quad b = \frac{\sqrt{c}}{c}$$

$$\lim_{x \rightarrow -\pi} \frac{f + k \left[\frac{x}{\pi} \right]}{\sin x} = +\infty$$

$$x > -\pi \Rightarrow \frac{x}{\pi} > -1 \rightarrow \left[\frac{x}{\pi} \right] = -1 \quad , \quad \sin(-\pi^+) = 0^+ \rightarrow f + k(-1) > 0 \rightarrow 2k < f \rightarrow k < \frac{f}{2} \quad (1)$$

$$x < -\pi \Rightarrow \frac{x}{\pi} < -1 \rightarrow \left[\frac{x}{\pi} \right] = -2 \quad , \quad \sin(-\pi^-) = 0^- \rightarrow f + k(-2) < 0 \rightarrow 2k > f \rightarrow k > \frac{f}{2} \quad (2)$$

$$(1) \& (2) \Rightarrow \frac{f}{2} < k < \frac{f}{2} \xrightarrow{x \rightarrow -1} -\frac{f}{2} < -k < -\frac{f}{2} \rightarrow \underline{[-k] = -\frac{f}{2}}$$

$$\lim_{x \rightarrow 1} \frac{b\sqrt{2x+3} - 2b}{ax - b} = \frac{2b - 2b}{1a - b} = \frac{0}{1a - b} \rightarrow \text{چون در صورت گفته شده صفر نیست پس باید رافع (L'Hôpital)}$$

$$\rightarrow \frac{b(\sqrt{2x+3} - 2)}{b(ax - 1)} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \frac{\sqrt{2x+3} - 2}{ax - 1} \times \frac{\sqrt{2x+3} + 2}{\sqrt{2x+3} + 2} = \frac{\sqrt{2x+3} - 2}{ax - 1} \times \frac{\sqrt{2x+3} + 2}{\sqrt{2x+3} + 2}$$

$$= \frac{x - 1}{4x - 4} = \frac{x - 1}{4(x - 1)} = \frac{1}{4}$$