

$$\lim_{x \rightarrow -r^+} f(x) = \lim_{x \rightarrow -r^-} f(x) \rightarrow \lim_{x \rightarrow -r^-} f(x) = r \left[\frac{r - (-r)}{r} \right] + a \left[\frac{(-r) + r}{r} \right] = r + a(-1) = r - a$$

$$\rightarrow \lim_{x \rightarrow -r^+} f(x) = r \left[\frac{r - (-r^+)}{r} \right] + a \left[\frac{(-r^+) + r}{r} \right] = r + a(0) = r$$

$$\Rightarrow r - a = r \Rightarrow a = 0 \rightarrow \left\{ \frac{a}{c} \right\} = \left\{ \frac{r}{c} \right\} = 0 \quad \checkmark$$

$$\lim_{x \rightarrow (\pi/4)^-} [r \sin x - 1] : r \sin \pi/4 - 1 = r(1/r) - 1 = 1 - 1 = 0 \rightarrow [0^-] = -1 \quad \checkmark$$

$$\frac{\pi}{4} = \pi/4 \rightarrow \sin \pi/4 = 1/r$$

(2)

$$\lim_{x \rightarrow -1/r} [r \ln x - x] : \left[r \ln \left(-\frac{1}{r} \right) - \left(-\frac{1}{r} \right) \right] = \left[-1 + \frac{1}{r} \right] = \left[-\frac{r-1}{r} \right] = -1 \quad \checkmark$$

در این جا باید به این نکته توجه کرد که $\ln$ فقط برای اعداد مثبت تعریف شده است.

$$\hookrightarrow \left[-\frac{1}{r} \right] = -1$$

(1)

$$\lim_{x \rightarrow (-1/r)^-} \frac{\ln x - a + \left[\frac{c}{x^r} \right]}{14x - \left[-\frac{r}{x^r} \right]} : \frac{1 \cdot \left(-\frac{1}{r} \right) - a + \left[\frac{r}{(-1/r)^r} \right]}{14 \left(-\frac{1}{r} \right) - \left[-\frac{r}{(-1/r)^r} \right]} = \frac{-10 + [12]}{-14 - [-1]} = \frac{-10 + 12}{-14 + 1} = \frac{2}{-13} = -\frac{2}{13}$$

(5)

$$\lim_{x \rightarrow \pi^+} \frac{\sin^r \pi x}{[x] + \cos \pi x} = \frac{(0)^r}{1 + (-1)} \rightarrow \frac{0}{0} \text{ صورت و مخرج}$$

$$\rightarrow \frac{\sin^r \pi}{1 + \cos \pi} \cdot \frac{1 - \cos \pi}{1 - \cos \pi} = \frac{\cancel{\sin^r \pi} (1 - \cos \pi)}{\cancel{1 - \cos \pi}} = 1 - \cos \pi = 1 - (-1) = 2 \quad \checkmark$$

(2)

$$\lim_{x \rightarrow 1^+} \frac{\sqrt{x+1} - \sqrt{x+1}}{1 + \sqrt{x}}$$

$$\text{hop} \rightarrow \frac{(1 \times \frac{1}{\sqrt{x+1}}) - (1 \times \frac{1}{\sqrt{x+1}})}{\frac{1}{2} \times \frac{1}{\sqrt{x+1}}} = \frac{(1) - (1)}{\frac{1}{2} \times 1} = \frac{0}{\frac{1}{2}} = 0$$

$$\lim_{x \rightarrow 1} \frac{x - \sqrt{x+1} + 2}{x - \sqrt{x+1}} \Rightarrow \frac{1 - (\frac{1}{2} \times \frac{1}{\sqrt{x+1}})}{1 - (\frac{1}{2} \times \frac{1}{\sqrt{x+1}})} = \frac{1 - \frac{1}{2}}{1 - \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$\lim_{x \rightarrow 0} \frac{a + \sqrt{bx+c}}{x} = \frac{1}{f} \xrightarrow{x=0} \frac{a + \sqrt{c}}{c} = \frac{0}{c} \Rightarrow a = -\sqrt{c}$$

$$\text{hop} \rightarrow \frac{bx \times \frac{1}{f} \times \frac{1}{\sqrt{bx+c}}}{1} = \frac{b}{f \sqrt{c}} = \frac{1}{f} \rightarrow \sqrt{b} = \sqrt{c} \rightarrow c = b$$

$$\lim_{x \rightarrow -\pi} \frac{f + k \left[\frac{x}{\pi} \right]}{\sin x} = +\infty \xrightarrow{x=-\pi} \frac{f + k \left[\frac{-\pi}{\pi} \right]}{0^+} = \frac{f - k}{0^+} = +\infty$$

(I) & (II) $\frac{f}{k} < k < 1 \rightarrow -k < k < -\frac{f}{k} \rightarrow [-k] = -1$

$\frac{f - k}{0^+} = +\infty$

$[k] = 1$

$$\lim_{x \rightarrow 1} \frac{b\sqrt{x+1} - 1}{x-1} = \frac{1b - 1}{1-1} \neq 0 \Rightarrow \frac{0}{0} \Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{x+1}}$$

سوال ۴

$$x \rightarrow \left(-\frac{1}{4}\right)^- \rightarrow x < -\frac{1}{4} \rightarrow x^2 > \frac{1}{4} \rightarrow \frac{1}{x^2} < 4 \rightarrow \frac{4}{x^2} < 16 \rightarrow \left[\frac{4}{x^2}\right] = 11$$

$$\frac{1}{x^2} < 4 \rightarrow -\frac{1}{x^2} > -4 \rightarrow \frac{-4}{x^2} > -16 \rightarrow \left[\frac{-4}{x^2}\right] = -11$$

$$\rightarrow \lim_{x \rightarrow \left(-\frac{1}{4}\right)^-} = \frac{10x - 20 + 11}{14x - (-1)} = \lim_{x \rightarrow \left(-\frac{1}{4}\right)^-} = \frac{10x + 9}{14x + 1} = \frac{-20 + 9}{(-1) + 1} = \frac{1}{0^-} = -\infty$$

سوال ۷

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(\sqrt{x} - 2)}{x - \sqrt{x^2 + 1}} = \frac{(\sqrt{x} - 1)(\sqrt{x} - 2)}{x - \sqrt{x^2 + 1}} \times \frac{x + \sqrt{x^2 + 1}}{x + \sqrt{x^2 + 1}} \rightarrow$$

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(\sqrt{x} - 2) \times (x + \sqrt{x^2 + 1})}{x^2 - x - 1} = \lim_{x \rightarrow 1} \frac{(\sqrt{x} - 1)(\sqrt{x} - 2)(x)}{(x+1)(x-1)} = \frac{(\sqrt{x}-1)(\sqrt{x}-2)(x)}{(x+1)(x-1)} \rightarrow \frac{(\sqrt{x}-1)(\sqrt{x}-2)(x)}{(x+1)(x-1)}$$

$$\lim_{x \rightarrow 1} \frac{(\sqrt{x} - 2)(x)}{(x+1)(\sqrt{x}+1)} = \frac{(-1)(1)}{2 \times 2} = -\frac{1}{4}$$

سوال ۱۰

$$\lim_{x \rightarrow 1} \frac{10(\sqrt{1+\sqrt{x}} - 1)}{ax - 10a} = \frac{10(\sqrt{1+\sqrt{x}} - 1)}{ax - 10a} \times \frac{\sqrt{1+\sqrt{x}} + 1}{\sqrt{1+\sqrt{x}} + 1} =$$

$$\lim_{x \rightarrow 1} \frac{10(\sqrt{x} - 1)}{a(x-1) \times 1} \rightarrow \lim_{x \rightarrow 1} \frac{1(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1 + 1 + \sqrt{x})} = \frac{1}{14} = \frac{1}{14}$$