نام و نام خانوادگی: حسام رفیع زاده « به نام خدا » مقطع: دوازدهم پسر A

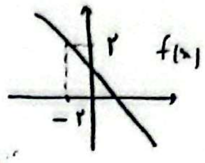
شماره: ۱۹

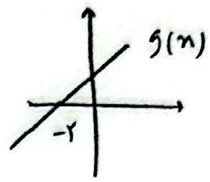
۱-

$$f(x) = 2\Big[\overbrace{\tfrac{2-x}{2}}^{f(x)}\Big] + a\Big[\overbrace{\tfrac{x+2}{3}}^{g(x)}\Big]$$

$f(x) = -\frac{1}{2}x + 1$

$g(x) = \frac{1}{3}x + \frac{2}{3}$





$$\lim_{x\to(-2)^+} 2[1] + a[0] = 2$$

$$\lim_{x\to(-2)^-} 2[2) + a[0^-] = 4 - a \quad \to \quad 4 - a = 2 \to \boxed{a = 2}$$

$$\left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] = 0$$

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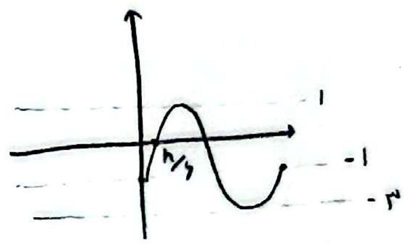
$$\lim_{x\to(\frac{\pi}{6})^-} [2\sin x - 1] = [0^-] = -1$$



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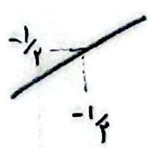
$$\lim_{x\to -\frac{1}{2}} [8x^3 - x] = \lim_{x\to(-\frac{1}{2})^+} = \left[(-\tfrac{1}{2})^+\right] = -1$$

با توجه به مطالب پایین

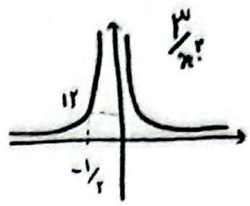
$$\lim_{x\to(-\frac{1}{2})^-} \left[(-\tfrac{1}{2})^-\right] = -1$$

رسم نمودار: $f'(x) = 24x^2 - 1 \to f'(-\frac{1}{2}) = 24 \times \frac{1}{4} - 1 = 5$ شیب مثبت

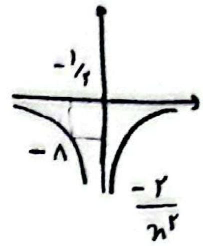


۴-

$$\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+\left[\frac{3}{x^2}\right]}{16x-\left[\frac{-2}{x^2}\right]}=\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+11}{16x-(-8)}=\frac{10x+6}{16x+8}$$



$$=\frac{-5+6}{-8+8}=\frac{\text{عدد}}{\text{صفر حدی}}\to\frac{-1}{0^-}=\boxed{+\infty}$$



۵-

$$\lim_{x\to1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\lim_{x\to1^+}\frac{\sin^2\pi x}{1+\cos\pi x}=\frac{1-\cos^2\pi x}{1+\cos\pi x}=\frac{(1-\cos\pi x)\cancel{(1+\cos\pi x)}}{\cancel{1+\cos\pi x}}$$

$$=\lim_{x\to1^+}1-\cos\pi x=\boxed{2}$$

۶-

$$\lim_{x\to-1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}=\frac{0}{0}\xrightarrow[\text{مزدوج}]{\text{ضرب در}}\frac{2x+3-(3x+4)}{(1+\sqrt[3]{x})(\sqrt{2x+3}+\sqrt{3x+4})}\xrightarrow{\text{ضرب در چاق و لاغر}}$$

$$\lim_{x\to-1}\frac{-\cancel{(x+1)}(1+\sqrt[3]{x^2}-\sqrt[3]{x})}{\cancel{(x+1)}(\sqrt{2x+3}+\sqrt{3x+4})}=\lim_{x\to-1}\frac{1+\sqrt[3]{x^2}-\sqrt[3]{x}}{\sqrt{2x+3}+\sqrt{3x+4}}=\boxed{\frac{-3}{2}}$$

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$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}=\frac{(\sqrt{x}-1)(2\sqrt{x}-5)}{2x-\sqrt{3x+1}}\xrightarrow{\times\text{مزدوج}}\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)(2x+\sqrt{3x+1})}{4x^2-3x-1}$$

$$=\lim_{x\to 1}\frac{(\sqrt{x}-1)(2\sqrt{x}-5)(2x+\sqrt{3x+1})}{(x-1)(4x+1)}=\lim_{x\to 1}\frac{\cancel{(\sqrt{x}-1)}(2\sqrt{x}-5)(2x+\sqrt{3x+1})}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)(4x+1)}\overset{\text{جایگذاری}}{=}$$

$$\lim_{x\to 1}\frac{-3\times \cancel{4}}{\cancel{2}\times 5}=\boxed{\frac{-6}{5}}$$

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$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}\xrightarrow[\frac{0}{0}\ \text{بوده}]{\text{این به صورت}}x=0\to a+\sqrt{b(0)+c}=0\to\sqrt{c}=-a\to c=a^2\quad ①$$

$a<0$ چون $\sqrt{c}$ مثبت

$$\xrightarrow{(H)}\lim_{x\to 0}\frac{\frac{b}{2\sqrt{bx+c}}}{1}=\frac{1}{4}\to\frac{b}{\cancel{2}\sqrt{c}}=\frac{1}{\cancel{4}_{2}}\to 2b=\sqrt{c}\to c=4b^2\quad ②\quad\to a^2=4b^2\to |a|=|2b|$$

$$\frac{ab}{c}\overset{①}{=}\frac{ab}{a^2}=\frac{b}{a}=\frac{\cancel{b}}{2\cancel{b}}=\boxed{\frac{1}{2}}$$

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$$\lim_{x\to -2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty$$

$$\to\lim_{x\to(-2\pi)^+}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{4-2k}{\sin x}=\lim_{x\to(-2\pi)^+}\frac{4-2k}{0^+}=+\infty\quad ①$$

$$\to\lim_{x\to(-2\pi)^-}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\lim_{x\to(-2\pi)^-}\frac{4-3k}{\sin x}=\lim_{x\to(-2\pi)^-}\frac{4-3k}{0^-}=+\infty\quad ②$$

$$\xrightarrow{①}4-2k>0\to k<2$$

$$\xrightarrow{②}4-3k<0\to 3k>4\to k>\frac{4}{3}\to\frac{4}{3}<k<2\xrightarrow{\times(-1)}-2<-k<\frac{-4}{3}\xrightarrow{[\,]}\boxed{[-k]=-2}$$

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$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}=\frac{2b-2b}{8a-b}=0$$ غ ق ق چون گفته صفر نیست $\longrightarrow \frac{0}{0} \rightarrow 8a-b=0 \longrightarrow 8a=b$ ①

$$\text{(H)} \longrightarrow \lim_{x\to 8}\frac{b\times\frac{1}{2\sqrt{2+\sqrt[3]{x}}}\times\frac{1}{3\sqrt[3]{x^2}}}{a}=\lim_{x\to 8}\frac{\frac{b}{4}\times\frac{1}{12}}{a}=\frac{b}{4\times 12\times a}\xrightarrow{①}\frac{\overset{2}{\cancel{8}}\cancel{a}}{\cancel{4}\times 12\times\cancel{a}}=\boxed{\frac{1}{6}}$$