دوازدهم سپهر / محمد عماد عطاریان (1)

-۱

$f(x)=x^2+2x+1+9x^2-6x+1+mx^2+nx+mn$

$\hookrightarrow x^2(1+9+m)+x(2-6+n)+mn+2$

$10+m=0$, $\boxed{m=-10}$ $\qquad -4+n=0$, $\boxed{n=4}$

$g(x)=\{(4,-10),(4,p+1),(p+2,-1),(-9,p+k)\}$

$-10=p+1$, $\boxed{p=-11}$ $\qquad (-9,-1)$, $-11+k=-1$, $\boxed{k=10}$

$\Rightarrow m+n+p+k=-10+4-11+10=\boxed{-7}$

-۲

الف) $x^4-2x^2-8\neq 0 \rightsquigarrow x^2=t \to t^2-2t-8\neq 0$

$(t-4)(t+2)\neq 0 \quad t=-2 \to x^2\neq -2$

$\hookrightarrow t=4 \to x^2=4 \to x\neq \pm 2$

$\dfrac{(x-2)}{(x-2)(x+2)(x^2+4)}\geq 0$ $\qquad$ $-2 \quad 2^*$: $-$ ن $+$ ن $+$

$x\neq 2$, $x\neq -2$, همواره مثبت

$\Rightarrow D_f=(-2,+\infty)-\{2\}$

ب) $4-2[-x]\neq 0 \to [-x]\neq 2$

$\Rightarrow D_f=\mathbb{R}-[-2,-1)$

-۳

الف) $|x|+x\neq 0 \to |x|\neq -x \to x\in(0,+\infty)$ Ⓐ

با فرض مثبت بودن مخرج $\Rightarrow \dfrac{x(x^2-1)}{2x}\geq 0$ $\qquad$ $-1 \quad 0 \quad 1$: $-$ $+$ ن $-$ $+$ Ⓑ

$\Rightarrow A\cap B=[1,+\infty)$

ب) $|x|-1\neq 0 \to x\neq \pm 1$ Ⓐ

$|x-2|-x^2\geq 0 \to x\geq 2 \to x^2-x+2\leq 0 \to$ همواره مثبت

$\hookrightarrow x\leq 2 \to x^2+x-2\leq 0$

$\hookrightarrow (x+2)(x-1)\leq 0$ $\qquad$ $-2 \quad 1$: $+$ $-$ $+$ Ⓑ

$A\cap B=[-2,1]-\{\pm 1\}$

-۴ اگر با حدس زدن k را برابر 1 بگیریم:

$||x-2|-1|-1\neq 0 \to |x-2|-1\neq 1 \to \boxed{x\neq 4,0}$

$|x-2|-1\neq -1 \to \boxed{x\neq 2}$

$\boxed{k=1}$ ⇐ چون ۳ عدد بدست آمد ⇐

-۵

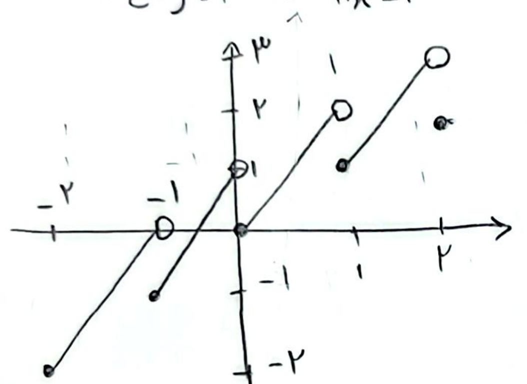
$-2\leq x<-1 \to [x]=-2 \to 2x+2$

$-1\leq x<0 \to [x]=-1 \to 2x+1$

$0\leq x<1 \to [x]=0 \to 2x$

$1\leq x<2 \to [x]=1 \to 2x-1$

$x=2 \to [x]=2 \to 2x-2$



-۶

همانی باشد $\to f(x)=x \to x-b=\dfrac{(x-3)(x-1)}{x+a}$

$\Rightarrow x^2+(a-b)x-ba=x^2-4x+3$

$\hookrightarrow a-b=-4$

$ab=-3 \to a=-\dfrac{3}{b}$ $\Big\}$ $-\dfrac{3}{b}-b=-4$

$\hookrightarrow b^2-4b+3=0$

$(b-3)(b-1)=0$

$a=-1$ یا -3 ⇐ $b=3$ یا $b=1$

$\Rightarrow a-b= -1-3=-4$; $-3-1=-4$ $\Big\}$ $\boxed{a-b=-4}$

-۷

$\dfrac{x^3+2x^2-x-2}{x^2-1} \Rightarrow x+2 \quad x\neq \pm 1$

$-x^3+x$

$2x^2-2$

$-2x^2+2$

$\hookrightarrow x=3 \to f(3)=5$

$g(3)=3+c=5 \Rightarrow \boxed{c=2}$

$\dfrac{ax+2}{x+b}$ $\quad x=1,\ g(1)=3$: $a+2=3b+3 \to a-3b=1$

$x=-1,\ g(-1)=1$: $-a+2=b-1 \to a+b=3$

$4b=2$, $\boxed{b=1/2}$

$b+c=\dfrac{1}{2}+2=\boxed{\dfrac{5}{2}}$

8-

$2(\sqrt{4-3a}+k-1)-(\sqrt{b-4}+3k)=k$

$2\sqrt{4-3a}+2k-2-\sqrt{b-4}-3k=k$

$2\sqrt{4-3a}-\sqrt{b-4}-2k-2=0$

$4-3a=0 \qquad b-4=0 \qquad -2k-2=0$

$a=4/3 \qquad b=4 \qquad k=-1$

$\Rightarrow 3(\frac{4}{3})-(4/2)-(-1)=4-2+1=\boxed{3}$

9-

$D_g = 2x+2 \geq 0 \rightarrow x \geq -1$

$g(3)=2\sqrt{2} \rightsquigarrow f(3)=8\sqrt{2}$ } $n=8\sqrt{2}-2$

$f(3)=2\sqrt{7-3}+n$

$D_{f \times g}=[-1,7]$

↳ $f(x)$ برای $\rightsquigarrow m-7=0 \rightarrow \boxed{m=7}$

$g(x)$ برای

$\Rightarrow m+n=7+8\sqrt{2}-2 \rightsquigarrow \boxed{8\sqrt{2}+5}$

10-

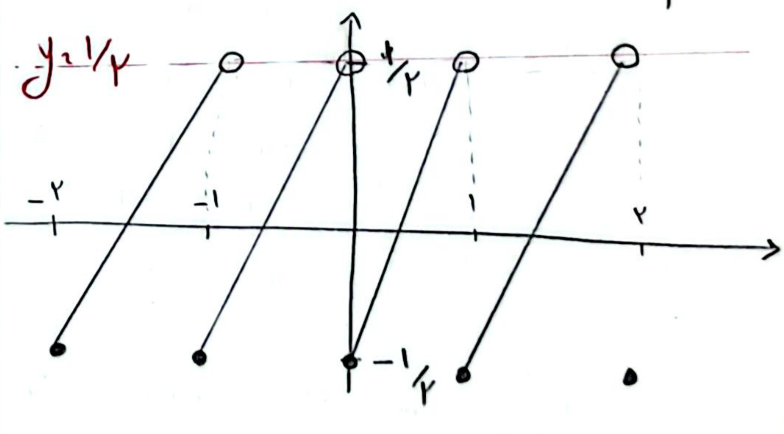
$-2 \leq x < -1 \rightarrow [x]=-2 \rightarrow x+2=1/2 \rightarrow x=-3/2$

$-1 \leq x < 0 \rightarrow [x]=-1 \rightarrow x+1=1/2 \rightarrow x+1/2$

$0 \leq x < 1 \rightarrow [x]=0 \rightarrow x=1/2 \rightarrow x-1/2$

$1 \leq x < 2 \rightarrow [x]=1 \rightarrow x-1=1/2 \rightarrow x-3/2$

$x=2 \rightarrow [x]=2 \rightarrow x-2=1/2 \rightarrow x-5/2$



ج-د) هیچ جوابی ندارد.