

$$f(x) = x^2 + 2x + 1 + 9x^2 - 6x + 1 + mx^2 + nx + mn$$

یعنی تابعی ثابت است پس مجموع ضرایب x^2 و x باید صفر شود

$$\Rightarrow f(x) = 10x^2 - 4x + mx^2 + nx + mn + 2 \Rightarrow m = -10 \text{ و } n = 4 \Rightarrow f(x) = -4x + 2$$

$$g(x) = \frac{1}{2} (4x - 1) (4x + 1) (4x^2 - 1) (4x^2 + 1) \Rightarrow 4x^2 - 1 = 0 \Rightarrow 4x^2 = 1 \Rightarrow x = \pm \frac{1}{2}$$

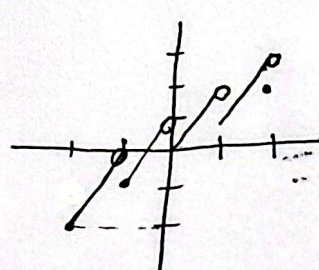
$$f(x) = \frac{x^2 - 2}{x^4 - 2x^2 - 1} \rightarrow \frac{x^2 - 2}{x^4 - 2x^2 - 1} \geq 0 \rightarrow \frac{x^2 - 2}{(x^2 - 1)(x^2 + 1)} \geq 0$$

$$\Rightarrow D_f = (-\infty, -1) \cup (1, \infty) \cup \{0\}$$

$$f(x) = \sqrt{x(x^2 - 1)} \rightarrow x(x^2 - 1) \geq 0 \rightarrow \frac{-1}{-x} \geq 0 \rightarrow D_1 = [-1, 0] \cup [1, \infty)$$

$$\Rightarrow D_f = D_1 \cap D_2 = [-1, 0] \cup [1, \infty)$$

$y = 2x - [x]$	$x = 2$	
$2x - 1$	$1 < x < 2$	
$2x$	$0 < x < 1$	
$2x + 1$	$-1 < x < 0$	
$2x + 2$	$-2 < x < -1$	



تابعی همانی است که برنویس از ناحیه اول و سوم منطبق باشد

$$f(x) = \frac{x^2 - 4x + 3}{x + a} + b$$

صورت باید با مخرج ساده شود

$$\frac{(x-3)(x-1)}{x+a} + b \Rightarrow a = -3 \Rightarrow \frac{(x-3)(x-1)}{x-3} + b$$

باید بشود $y=x$

$$\rightarrow x-1+b=y \Rightarrow b=1$$

$a=-1 \Rightarrow \frac{(x-3)(x-1)}{x-1} + b \rightarrow x-3+b=y$

$a=-3 \Rightarrow a-b = -3-1 = -4$

$a=-1 \Rightarrow a-b = -1-3 = -4$

$$f(x) = \begin{cases} \frac{x^3 + 4x^2 - x - 2}{x^2 - 1} ; x \neq \pm 1 \\ \frac{ax + 2}{x + b} ; x = \pm 1 \end{cases}$$

$$g(x) = \frac{ax + 2}{x + b}$$

$x=1 \rightarrow \frac{a+2}{1+b} = 1+c \Rightarrow a+2 = 1+b+c \Rightarrow a+b+c = 1$

$x=-1 \Rightarrow \frac{-a+2}{-1+b} = -1+c \Rightarrow -a+2 = -1+c-b \Rightarrow -a+b-c = -3$

$\begin{cases} a+b+c = 1 \\ -a+b-c = -3 \end{cases} \Rightarrow \begin{cases} a+b+c = 1 \\ a+b-c = 3 \end{cases} \Rightarrow \begin{cases} 2c = -2 \Rightarrow c = -1 \\ b+c = 1-1 = 0 \Rightarrow b = 1 \end{cases}$

$\frac{-2}{-1} = c \Rightarrow c = 2 \Rightarrow b = \frac{1}{2} \Rightarrow b+c = \frac{1}{2} + 2 = \frac{5}{2}$

$2f - g = \sum (1, 9, 1) \Rightarrow$ اشتراک دامنه های f و g است

تنها حالت برقراری شرط بالا است

$$\Rightarrow \frac{b}{\varepsilon} = 1 = \frac{3}{\varepsilon} a \Rightarrow b = \varepsilon a \Rightarrow 3a = \varepsilon$$

ادامه: $2f(1) - g(1) = \sum (1, 9, k)$

$$\Rightarrow 2\sqrt{\varepsilon - b} - 2 - \sqrt{b - \varepsilon} - 3k = k \Rightarrow 2k - 2 - 3k = k \Rightarrow 2k = -2 \Rightarrow k = -1$$

$\Rightarrow 3a - \frac{b}{\varepsilon} - k = \varepsilon - 2 + 1 = 3$

اشتراک دو دامنه

$$D_{f \cap g} = [-1, 9] \Rightarrow D_g: 2m + 2 \geq 0 \Rightarrow 2m \geq -2 \Rightarrow m \geq -1$$

$\Rightarrow D_f = (-\infty, 9] \Rightarrow m - n \geq 0 \Rightarrow m \geq n \Rightarrow m = 9$

$f(3) - g(3) = 4\sqrt{2} \xrightarrow{g(3) = 1\sqrt{2}} f(3) - 1\sqrt{2} = 4\sqrt{2} \Rightarrow f(3) = 5\sqrt{2}$

$\Rightarrow f(3) = \sqrt{9 - 3} + n = 5\sqrt{2} \Rightarrow n = 5\sqrt{2} - 2 \Rightarrow m + n = 9 + 5\sqrt{2} - 2 = 7 + 5\sqrt{2}$

$$y = (-1)^n (x - [x]) = \frac{1}{y} \Rightarrow y = x - [x] = \frac{1}{y}$$

