

$$f(x) = (x+1)^2 + (2x-1)^2 + mx^2 + nx + mn$$

$$= x^2 + 1 + 2x + 4x^2 + 1 - 2x + mx^2 + mn + mn = (1+m)x^2 + (n-2)x + 2+mn$$

$$1+m=0 \rightarrow m=-1, \quad n-2=0 \rightarrow n=2$$

$$m+n+p+k = 1-1+2-1 = -1$$

$$g(x) = \left\{ (4, m), (n, p+1), (p+2, -1), (-9, p+k) \right\}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow & \downarrow \\ (4, -1) & (2, p+1) & (p+2, -1) & (-9, p+k) \end{matrix}$$

$$\rightarrow p+1 = -1 \rightarrow p = -2$$

$$\rightarrow -1 = -2 + k \rightarrow k = 1$$

الف) $y = \frac{x-2}{x^2-2x-1} \rightarrow \frac{x-2}{x^2-2x-1} \geq 0$

$$x^2-2x-1 = 0 \rightarrow x = 1 \pm \sqrt{2}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$$D_f = (-\infty, 1-\sqrt{2}) \cup (1+\sqrt{2}, +\infty)$$

ب) $f(x) = \frac{mx^2 + 1}{x-2[-x]}$

$$D_f = \mathbb{R} - [-2, -3]$$

الف) $f(x) = \frac{\sqrt{x(x^2-1)}}{\sqrt{|x|+x}}$

$$\rightarrow x(x^2-1) \geq 0$$

$$\rightarrow |x|+x > 0$$

$$x \in (0, +\infty) \cap [1, +\infty) = [1, +\infty)$$

ب) $f(x) = \frac{\sqrt{|x-2|-x^2}}{\sqrt{|x|-1}}$

$$\rightarrow |x-2|-x^2 \geq 0$$

$$|x|-1 \geq 0 \rightarrow x \in (-\infty, -1] \cup [1, +\infty)$$

$y = \frac{1}{||x-2|-1|-k}$

$$\rightarrow ||x-2|-1|-k \neq 0$$

$$\rightarrow ||x-2|-1|-k = 0$$

$$||x-2|-1| = k$$

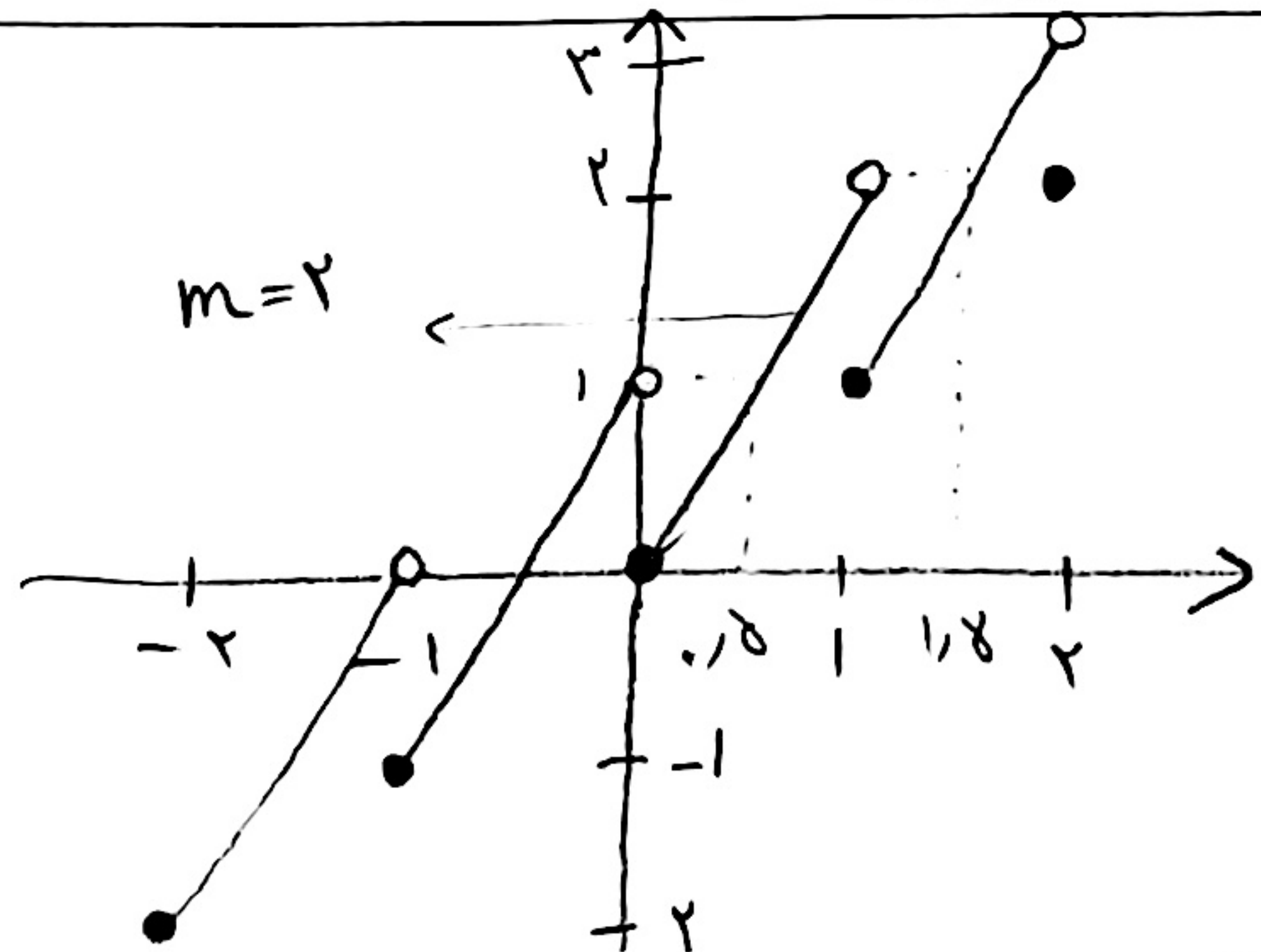
$$|x-2|-1 = k \rightarrow |x-2| = k+1$$

$$|x-2|-1 = -k \rightarrow |x-2| = 1-k$$

$$x-2 = k+1 \rightarrow x = k+3$$

$$x-2 = 1-k \rightarrow x = k+3$$

$$x-2 = k-1 \rightarrow x = k+1$$



$y = 2x - [x] \rightarrow 2x - p$

بازای x های غیر صحیح تابع به صورت خطی با شیب ثابت ۲ است و در نقاط صحیح x تابع دچار نا پیوستگی می شود.

$f(-2) = -2$
 $f(-1) = -1$
 $f(0) = 0$
 $f(1) = 1$
 $f(2) = 2$

$f(0.5) = 1 - [0.5] = 1$
 $f(1.5) = 3 - [1.5] = 2$

$$f(x) = \frac{x^2 - 2x + 3}{x+a} + b = \frac{(x-1)(x-3)}{x+a} + b = x \leftarrow \text{باقی ماند}$$

$$\text{if } (x+a) = (x-1), \{1\} \notin Df \quad (x-3) + b = x \rightarrow b = 3, \quad a = -1$$

$$\text{if } (x-3) = (x+a), \{3\} \notin Df \quad (x-1) + b = x \rightarrow b = 1, \quad a = -3$$

$$a-b \begin{cases} -1-3 = -4 \\ -3-1 = -4 \end{cases} \rightarrow -1-\{3\} \neq a-b \quad \text{در این دو مورد}$$

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$$f(x) \begin{cases} \frac{x^3 + 2x^2 - x - 2}{x^2 - 1}; x \neq \pm 1 \rightarrow \frac{(x-1)(x^2 + 2x + 2)}{(x+1)(x-1)} = \frac{(x-1)(x+1)(x+2)}{(x+1)(x-1)} = x+2 \\ \frac{ax+2}{x+b}; x = \pm 1 \end{cases} \quad g(x) = x+c \rightarrow c=2$$

$$\begin{aligned} f(1) = g(1) = +3 &\rightarrow \frac{a+2}{1+b} = 3 \rightarrow 3+3b = a+2 \rightarrow 3b-a = -1 \quad \text{if } b = 2 \\ f(-1) = g(-1) = +1 &\rightarrow \frac{-a+2}{b-1} = 1 \rightarrow b-1 = 2-a \rightarrow b+a = 3 \rightarrow \begin{cases} b = \frac{1}{3} \\ a = \frac{8}{3} \end{cases} \end{aligned}$$

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$$\begin{aligned} f-g = \{(1, k)\} \quad Df \cap Dg = \{1\} \quad b - \{x \geq 0\} \rightarrow \frac{b}{x} \geq x \quad Dg = (-\infty, \frac{b}{x}] \\ Df \cap Dg = \{1\} = \frac{b}{x} = \frac{3a}{x} \rightarrow b = 3a = 4 \quad \text{if } x - 3a \geq 0 \rightarrow \frac{3a}{x} \leq x \quad Df = [\frac{3a}{x}, +\infty) \end{aligned}$$

$$\begin{aligned} g(x) = \sqrt{4-4x} + 3k, \quad f(x) = \sqrt{4x-3a} + k - 1 \rightarrow (f-g)(1) \quad (1, k) \\ = 2(\sqrt{4-4} + k - 1) - (\sqrt{4-4} + 3k) = 2k - 2 - 3k = -k - 2 \rightarrow -k - 2 = k \\ 2k = -2 \rightarrow k = -1 \quad 3a - \frac{b}{x} - k = 4 - \frac{4}{1} - (-1) = 3 \end{aligned}$$

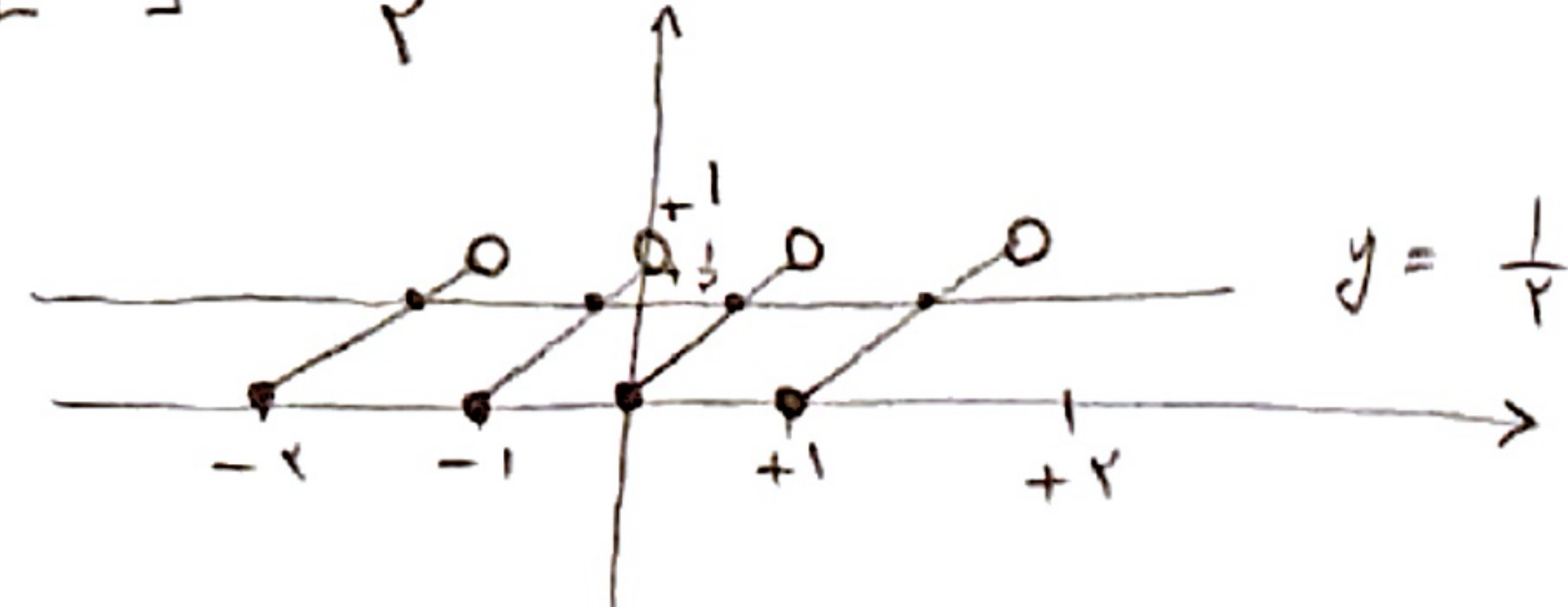
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$$\begin{aligned} Df, g = [-1, v] \rightarrow Df \cap Dg = [-1, v] \quad Df = (-\infty, m] \quad m - x \geq 0 \rightarrow x \leq m \\ Df \cap Dg = [-1, m] = [-1, v] \rightarrow m = v \quad Dg = [-1, +\infty) \quad 2x + 2 \geq 0 \rightarrow 2x \geq -2 \\ x \geq -1 \\ (f-g)(x) = 4\sqrt{2} \rightarrow \sqrt{v-3} + n - \sqrt{2(x)+2} = 4\sqrt{2} \rightarrow 2+n-2\sqrt{2} = 4\sqrt{2} \\ n = 1\sqrt{2} + 2 \\ m+n = 1\sqrt{2} + 2 + v = 1\sqrt{2} + 8 \end{aligned}$$

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$$\begin{aligned} y = (-1)^x (x - [x]) = \frac{1}{x} \\ = x - [x] = \frac{1}{x} \end{aligned}$$

خط $y = \frac{1}{x}$ تابع مور، تکرار، نقطه طغیانی اند.
(۲ جواب دارد)



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