

$\lim_{x \rightarrow 2} \frac{f(x) - a}{x^2 + ax + b} \xrightarrow{x=2} \frac{f(2) - a}{4 + 2a + b} = \frac{0}{0} \Rightarrow \frac{f'(2) - a}{2 + a} = 0 \quad (1)$
 $\Rightarrow f'(2) - a = 0 \Rightarrow \frac{f'(2)}{2 + a} = 0 \Rightarrow f'(2) = 0$
 $f'(x) = 2x + 1 \Rightarrow f'(2) = 5 \Rightarrow 5 = 0$ (contradiction)
 $\Rightarrow$ No solution for a, b such that the limit exists and is finite. (2)

$\lim_{x \rightarrow \sqrt{e}} \frac{f(x) - a}{x^2 + \sqrt{e}x + b} \xrightarrow{x=\sqrt{e}} \frac{f(\sqrt{e}) - a}{e + \sqrt{e} + b} = \frac{0}{0} \Rightarrow \frac{f'(\sqrt{e}) - a}{2\sqrt{e} + \sqrt{e} + b} = 0 \quad (1)$
 $\Rightarrow f'(\sqrt{e}) - a = 0 \Rightarrow \frac{f'(\sqrt{e})}{3\sqrt{e} + b} = 0 \Rightarrow f'(\sqrt{e}) = 0$
 $f'(x) = 2x \Rightarrow f'(\sqrt{e}) = 2\sqrt{e} \Rightarrow 2\sqrt{e} = 0$ (contradiction)
 $\Rightarrow$ No solution for a, b such that the limit exists and is finite. (2)

$\lim_{x \rightarrow -1} \frac{f(x) + a}{\sqrt{x} - x + a} = -\infty \Rightarrow \frac{f(-1) + a}{-1 - 1 + a} = -\infty \Rightarrow \frac{f'(-1) + a}{-2 + a} = -\infty$
 $\Rightarrow f'(-1) + a = 0 \Rightarrow \frac{f'(-1)}{a - 2} = 0 \Rightarrow f'(-1) = 0$
 $f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(-1) = -\frac{1}{2} \Rightarrow -\frac{1}{2} = 0$ (contradiction)
 $\Rightarrow$ No solution for a such that the limit exists and is finite. (2)

$\lim_{x \rightarrow -\frac{1}{2}} \frac{f(x) - \frac{1}{2}}{x^2 + \frac{1}{2}x} = \frac{f(-\frac{1}{2}) - \frac{1}{2}}{\frac{1}{4} - \frac{1}{4}} = \frac{0}{0} \Rightarrow \frac{f'(-\frac{1}{2}) - \frac{1}{2}}{-\frac{1}{2} + \frac{1}{2}} = \frac{f'(-\frac{1}{2}) - \frac{1}{2}}{0}$
 $\Rightarrow$ The limit does not exist (vertical asymptote). (2)

$\lim_{x \rightarrow 1} \frac{f(x) - 1}{x^2 - 1} = \frac{f(1) - 1}{1 - 1} = \frac{0}{0} \Rightarrow \frac{f'(1) - 1}{2x} = \frac{f'(1) - 1}{2} = \frac{1}{2}$
 $\Rightarrow f'(1) - 1 = 1 \Rightarrow f'(1) = 2$
 $f'(x) = 2x \Rightarrow f'(1) = 2$ (satisfies the condition)
 $\Rightarrow$ The limit exists and is $\frac{1}{2}$. (2)

$$\frac{x^2 + 10x + 14}{12 + 4\sqrt{x}} \Rightarrow x \rightarrow -1 \Rightarrow \frac{(x+2)(x+1)}{4(1+\sqrt{x})} \times \frac{\sqrt{x}-1}{\sqrt{x}-1} \Rightarrow \frac{(x+2)(x+1)(\sqrt{x}-1)}{4(1+\sqrt{x})(\sqrt{x}-1)}$$

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$$\frac{(x+2)(\sqrt{x}-1+\sqrt{x}+1)}{4} \Rightarrow x \rightarrow -1 \Rightarrow \frac{(-4)(1)}{4} = -1$$

$$\frac{\sqrt{x^2+x} - \sqrt{x-x}}{\sqrt{1-\cos x}} \Rightarrow \frac{\sqrt{x^2+x} + \sqrt{x-x}}{\sqrt{x^2+x} + \sqrt{x-x}} \Rightarrow \frac{x}{\sqrt{x^2+x}} \Rightarrow \frac{x}{x\sqrt{1+\frac{1}{x}}} \Rightarrow \frac{1}{\sqrt{1+\frac{1}{x}}} \Rightarrow \frac{1}{\sqrt{2}}$$

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$$\Rightarrow -\frac{1}{\sqrt{2}} \lim_{x \rightarrow 0^-} \frac{\sqrt{x^2+x} - \sqrt{x-x}}{\sqrt{1-\cos x}} \times \frac{\sqrt{x^2+x} + \sqrt{x-x}}{\sqrt{x^2+x} + \sqrt{x-x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \Rightarrow \lim_{x \rightarrow 0^-} \frac{x^2+x-x}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{x\sqrt{2}} = \lim_{x \rightarrow 0^-} \frac{x^2}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{x\sqrt{2}} = \lim_{x \rightarrow 0^-} \frac{x}{\sqrt{1-\cos x}} \times \frac{\sqrt{1+\cos x}}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\frac{k + \cos(\sqrt{x})}{kx^2} \Rightarrow k + \cos(0) = k + 1 = 0 \Rightarrow k = -1$$

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$$\text{L'Hôpital} \Rightarrow \frac{-1 + \cos \sqrt{x}}{-2x} \Rightarrow \frac{-\sin \sqrt{x} \cdot \frac{1}{2\sqrt{x}}}{-2x} \Rightarrow \frac{\sin \sqrt{x}}{4x\sqrt{x}} \Rightarrow \frac{\sin \sqrt{x}}{4x^{3/2}} \Rightarrow \frac{1}{4}$$

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$$\frac{\sqrt{x^2-9a^2} + \sqrt{x^2-9a^2}}{\sqrt{x^2-9a^2}} \Rightarrow \frac{\sqrt{x^2-9a^2} + \sqrt{x^2-9a^2}}{\sqrt{x^2-9a^2}} \times \frac{\sqrt{x^2+9a^2}}{\sqrt{x^2+9a^2}} \Rightarrow \frac{x^2-9a^2 + x^2+9a^2}{(x^2-9a^2)\sqrt{x^2+9a^2}} \Rightarrow \frac{2x^2}{(x^2-9a^2)\sqrt{x^2+9a^2}}$$

$$x \rightarrow 9a^2 \Rightarrow \frac{1}{\sqrt{x^2-9a^2}} + 0 \Rightarrow \frac{1}{\sqrt{9a^2-9a^2}} \Rightarrow \frac{1}{0} \Rightarrow \infty$$

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$$\frac{1-k[x]}{(x-1)(x+1)} \Rightarrow \frac{1+k}{(x+1)(x+1)} \Rightarrow 1+k > 0 \Rightarrow k > -1$$

$$\Rightarrow \frac{1+k}{(x-1)(x+1)} \Rightarrow 1+k < 0 \Rightarrow k < -1$$

$$\frac{1-k[x]}{(x-1)(x+1)} \Rightarrow \frac{1-k}{(x-1)(x+1)} \Rightarrow \frac{1-k}{(x-1)(x+1)} \Rightarrow \frac{1-k}{(x-1)(x+1)} \Rightarrow \frac{1-k}{(x-1)(x+1)}$$

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