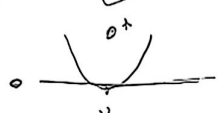


نام و نام خانوادگی: کلاس: شماره: پاسخنانه تشریحی تکلیف شماره: ۱۸

$$\lim_{x \rightarrow 1} \frac{x^2 - 9}{x^2 + ax + b} = \frac{-1}{x^2 + ax + b} = \frac{-1}{x^2 - 2x + 1} = \frac{-1}{(x-1)^2} = \frac{-1}{0^+} = -\infty$$

$$\lim_{x \rightarrow 1} \underbrace{x^2 + ax + b}_{0^+} = 0 \Rightarrow (x-1)^2 = x^2 - 2x + 1 \Rightarrow a + b = 0 \quad \checkmark$$


۱

(۲)

$$\lim_{x \rightarrow \sqrt[3]{\varepsilon}} \frac{\sqrt[3]{\varepsilon}}{x^2 + ax + b} = +\infty$$

$$\Rightarrow x^2 + ax + b = 0 \Rightarrow (x - \sqrt[3]{\varepsilon})^2 = x^2 - 2\sqrt[3]{\varepsilon}x + \sqrt[3]{\varepsilon}^2$$

$$\Rightarrow \frac{b}{a} = \frac{-\sqrt[3]{\varepsilon}^2}{-2\sqrt[3]{\varepsilon}} = \frac{1}{2} \Rightarrow \left[\frac{1}{2} \right] = \left[-\frac{1}{2} \right] = -\frac{1}{2} \quad \checkmark$$

$$\sqrt[3]{\varepsilon} = 1, \dots$$

۲

(۲)

$$\frac{1}{x^2} = 0^+ \Rightarrow \left[-\frac{1}{\sqrt[3]{x}} \right] = -1 \Rightarrow \lim_{x \rightarrow \frac{1}{\sqrt[3]{x}}} \frac{a-1}{x^2 + ax + b} = -\infty$$

(۲)

۳

$$\lim_{x \rightarrow \frac{1}{\sqrt[3]{x}}} \frac{-\frac{1}{x}}{x^2 + ax + b} = -\infty$$

$$\left(\frac{1}{\sqrt[3]{x}} \right)^2 + \left(\frac{1}{\sqrt[3]{x}} \right) + a = 0 \Rightarrow \left[a \right] = \left[-\frac{1}{4} \right] = -\frac{1}{4} \quad \checkmark$$

$$-a = \frac{1}{x} + a \quad \text{or} \quad a = \frac{1}{x} + a \quad \checkmark$$

$$-\frac{1}{x} \rightarrow \frac{x^2}{x^2} \rightarrow \frac{x^2}{x^2} \rightarrow -\frac{x^2}{x^2} \Rightarrow 14x - \left[-\frac{x^2}{x^2} \right] = 14x - [-1] = 14x + 9$$

$$-\frac{1}{x} \rightarrow \frac{x^2}{x^2} \rightarrow \frac{x^2}{x^2} \Rightarrow 14x + \left[\frac{x^2}{x^2} \right] = 14x + [1] = 14x + 1$$

$$\Rightarrow \frac{14x + 9}{14x + 1} = \frac{1}{0^+} = +\infty \quad \checkmark$$

(۲)

۴

$$\lim_{x \rightarrow 1} \frac{x^2 - 9}{x^2 - 1} = \frac{x^2 - 9}{x^2 - 1} = \frac{(x-3)(x+3)}{(x-1)(x^2 + x + 1)} = \frac{2}{16} = \frac{1}{8} \quad \checkmark$$

(۲)

۵

$$\lim_{x \rightarrow -1} \frac{x^2 + \ln x + 1}{1 + 4\sqrt{x}} = \frac{(x+1)(x+1)}{4(1+\sqrt{x})} \cdot \frac{(x+1)(\sqrt{x}+1)(\sqrt{x}+1+\epsilon)}{4(\sqrt{x}+1)} \\ = \frac{(-4)(\epsilon + \epsilon + \epsilon)}{4} = -1 \quad \checkmark$$

6

$$\frac{\sqrt{1+x} - \sqrt{1-x}}{1 - \cos x} = \frac{\sqrt{1+x} - \sqrt{1-x}}{1 - \cos x} \cdot \frac{1}{1} = \frac{1 + 1x - 1 + x}{-2x \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}} = \frac{\epsilon x}{-2x} = -\frac{1}{2}$$

7

$$\lim_{x \rightarrow 0} \frac{k + \cos(\sqrt{a}x)}{kx^2} \xrightarrow{\text{Hop}} \frac{-\sqrt{a} \sin(\sqrt{a}x)}{2kx} = \frac{-a}{2k} = -\frac{a}{2k} = 1 \\ \Rightarrow \frac{a}{k} = -4 \quad \checkmark$$

8

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x-a}}{\sqrt{x-a}\sqrt{x+a}} + \frac{\sqrt{x-a}\sqrt{x+a}}{\sqrt{x-a}\sqrt{x+a}} = \lim_{x \rightarrow \infty} \left(\frac{1}{\sqrt{a}} + \frac{\sqrt{x-a}\sqrt{x+a}}{\sqrt{x-a}\sqrt{x+a}} \right) \rightarrow \\ \lim_{x \rightarrow \infty} \left(\frac{1}{\sqrt{a}} + \frac{1(x-a)}{(\sqrt{x-a}\sqrt{x+a})(\sqrt{x+a})} \right) = \lim_{x \rightarrow \infty} \left(\frac{1}{\sqrt{a}} + \frac{1\sqrt{x-a}}{(\sqrt{x-a}\sqrt{x+a})(\sqrt{x+a})} \right) = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{a}} = 1 \\ \rightarrow \frac{\sqrt{1}}{\sqrt{a}} = \sqrt{\frac{1}{a}} = \sqrt{\frac{1}{a}}$$

9

$$\begin{aligned} -1^+ \rightarrow \frac{1+k}{(x-1)(x+1)} &= \frac{1+k}{0^-} = -\infty \Rightarrow 1+k > 0 \quad k > -1 \\ -1^- \rightarrow \frac{1+k}{x^2-1} &= -\infty \Rightarrow 1+k < 0 \quad k < -1 \end{aligned}$$

Unit Circle

Unit Circle

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