

$$\lim_{x \rightarrow 2} \frac{x^2 - 5}{x^2 + ax + b} = \lim_{x \rightarrow 2} \frac{-1}{4 + 2a + b} = -\infty \rightarrow 2a + b + 4 = 0 \rightarrow 2a + b = -4$$

$$x^2 + ax + b = (x + \frac{a}{2})^2 = (x - 2)^2 = x^2 - 4x + 4 \rightarrow \begin{cases} a = -4 \\ b = 4 \end{cases}$$

$$a + b = -4 + 4 = 0$$

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$$\lim_{x \rightarrow \sqrt[3]{4}} \frac{x}{x^2 + ax + b} = \lim_{x \rightarrow \sqrt[3]{4}} \frac{\sqrt[3]{4}}{\sqrt[3]{16} + \sqrt[3]{4}a + b} = -\infty \rightarrow \sqrt[3]{4}a + b + \sqrt[3]{16} = 0 \rightarrow \sqrt[3]{4}a + b = -\sqrt[3]{16}$$

$$x^2 + ax + b = (x + \frac{a}{2})^2 = (x - \sqrt[3]{4})^2 = x^2 - 2\sqrt[3]{4}x + \sqrt[3]{16} \rightarrow \begin{cases} a = -2\sqrt[3]{4} \\ b = \sqrt[3]{16} = 2\sqrt[3]{2} \end{cases}$$

$$\left[\frac{b}{a}\right] = \left[\frac{2\sqrt[3]{2}}{-2\sqrt[3]{4}}\right] = \left[\frac{1}{-\sqrt[3]{2}}\right] = \left[-\frac{\sqrt[3]{4}}{\sqrt[3]{2}}\right] = -1$$

$$1 < \sqrt[3]{4} < 2 \rightarrow \frac{1}{2} < \frac{\sqrt[3]{4}}{2} < 1 \rightarrow -1 < -\frac{\sqrt[3]{4}}{2} < -\frac{1}{2}$$

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$$\lim_{x \rightarrow (\frac{1}{p})^+} \frac{[-x] + a}{|\frac{1}{p}x + a| + a} = \frac{a-1}{|\frac{1}{p}x + a| + a} = \frac{a-1}{|\frac{1}{p} + a| + a} = -\infty \rightarrow |a + \frac{1}{p}| + a = 0$$

$$a \geq \frac{1}{p} \quad 2a + \frac{1}{p} = 0 \rightarrow a = -\frac{1}{2p} \rightarrow \frac{-\frac{1}{2p}}{0^+} = -\infty \quad \checkmark \rightarrow a = -\frac{1}{2p}$$

$$a < \frac{1}{p} \quad -a - \frac{1}{p} + a = 0 \rightarrow -\frac{1}{p} = 0 \quad \text{QOE}$$

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$$\lim_{x \rightarrow (\frac{1}{2})^+} \frac{14x - [\frac{2}{x^2}]}{24x + [\frac{2}{x^2}]} = \lim_{x \rightarrow (\frac{1}{2})^+} \frac{14x + 4}{24x + 12} = \frac{1}{0^+} = +\infty$$

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$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^2 - [x^2]} = \frac{x^2 - 4}{x^2 - 1} = \frac{(x-2)(x+2)}{(x-1)(x^2 + 2x + 4)} = \frac{x+2}{x^2 + 2x + 4} = \frac{4}{12} = \frac{1}{3}$$

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$$\lim_{x \rightarrow -1} \frac{x^2 + 10x + 12}{9\sqrt{x} + 12} = \frac{(x+2)(x+6)}{9(\sqrt{x}+2)} = \frac{(\sqrt{x}^2 + 2\sqrt{x} + 2)(\sqrt{x}+2)(x+2)}{9(\sqrt{x}+2)} = \frac{(\sqrt{x}^2 + 2\sqrt{x} + 2)(x+2)}{9}$$

$$= \frac{5 \times (-5)}{9} = \boxed{-\frac{25}{9}}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1-5x}} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} = \frac{2x}{\sqrt{1-5x} \times 2\sqrt{1-x}} = \frac{x}{|x| \times \sqrt{1-x}} = \boxed{-1}$$

$$\cos x \sim 1 - \frac{x^2}{2} \rightarrow 1 - 5x \sim \frac{x^2}{2}$$

$$\lim_{x \rightarrow 0} \frac{k + \cos(\sqrt{x})}{kx^2} = \frac{k + \cos 0}{0} = \frac{1+k}{0} = \infty \rightarrow \frac{0}{0} \rightarrow k+1=0 \rightarrow k=-1$$

$$\lim_{x \rightarrow 0} \frac{\cos(\sqrt{x}) - 1}{-x^2} \xrightarrow{\text{h.o.p.}} \frac{-\sqrt{x} \sin(\sqrt{x})}{-2x} = \frac{0}{0} \xrightarrow{\text{h.o.p.}} \frac{-\frac{1}{2} \cos(\sqrt{x})}{-1} = \frac{-\frac{1}{2}}{-1} = \frac{1}{2} = \infty \rightarrow a = \frac{1}{2}$$

$$\frac{a}{k} = \frac{\frac{1}{2}}{-1} = \boxed{-\frac{1}{2}}$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-xa} + \sqrt{x} - \sqrt{xa}}{\sqrt{x^2-9a^2}} = \frac{0}{0} \rightarrow \lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-xa}}{\sqrt{x^2-9a^2}} + \frac{\sqrt{x} - \sqrt{xa}}{\sqrt{x^2-9a^2}}$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-xa}}{\sqrt{x^2-9a^2}} = \frac{\sqrt{a}}{\sqrt{4a}}$$

$$\lim_{x \rightarrow (\sqrt{a})^+} \frac{\sqrt{x-xa}}{\sqrt{x^2-9a^2}} \times \frac{\sqrt{x} + \sqrt{xa}}{\sqrt{x} + \sqrt{xa}} = \frac{\sqrt{x-xa}}{\sqrt{x+3a} \times \sqrt{x-3a} \times \sqrt{xa}} = \frac{\sqrt{x-xa}}{\sqrt{x+3a} \times \sqrt{xa}} = 0$$

$$\frac{\sqrt{a}}{\sqrt{4a}} + 0 = \frac{\sqrt{a}}{\sqrt{4a}}$$

$$\lim_{x \rightarrow (-1)} \frac{1-k[x]}{x^2-1} \xrightarrow{(+)} \frac{1+k}{x^2-1} = \frac{1+k}{0^-} = -\infty \rightarrow 1+k > 0 \rightarrow k > -1$$

$$\xrightarrow{(-)} \frac{1+k}{x^2-1} = \frac{1+k}{0^+} = -\infty \rightarrow 1+k < 0 \rightarrow k < -1$$

$$-1 < k < -\frac{1}{2} \rightarrow -\pi < k\pi < -\frac{\pi}{2} \rightarrow x < 0$$

$$-1 < \cos k\pi < 0 \rightarrow y < 0 \rightarrow \boxed{(-\pi, -\frac{\pi}{2})}$$