

با توجه به اینکه صورت با عدد ۲ منفی می شود پس
عدد دو باید مخرج را صفر کند و مخرج توان دو باشد

$$\lim_{x \rightarrow 2} \frac{2x - 4}{x^2 + ax + b} = -\infty$$

$$\downarrow$$

$$c(x-2)^2 = x^2 + 4 - 4x \rightarrow b = 4, a = -4 \quad a+b = 4-4 = 0$$

از آنجایی که صورت + است پس مخرج هم باید صفر شود

$$\lim_{x \rightarrow \sqrt[3]{4}} \frac{x}{x^3 + ax + b} = +\infty$$

$$\downarrow$$

$$c(x - \sqrt[3]{4})^2$$

$$x^3 + \sqrt[3]{16} - 2x\sqrt[3]{4} \rightarrow a = -2\sqrt[3]{4}, b = \sqrt[3]{16} = \sqrt[3]{4} \times \sqrt[3]{4}$$

$$\left[\frac{b}{a} \right] = \left[\frac{\sqrt[3]{4} \times \sqrt[3]{4}}{-2\sqrt[3]{4}} \right] = \left[-\frac{\sqrt[3]{4}}{2} \right]$$

$$= [-0/\dots] = -\frac{1}{2}$$

وقتی جواب حدین برابر شود
پس مخرج صفر شود

$$\lim_{x \rightarrow \frac{1}{\sqrt{3}}} \frac{[-x] + a}{\left| \frac{1}{\sqrt{3}}x + a \right| + a} = -\infty \rightarrow \lim_{x \rightarrow \frac{1}{\sqrt{3}}} \frac{-1+a}{\left| \frac{1}{\sqrt{3}} + a \right| + a} = -\infty$$

$$\left| \frac{1}{\sqrt{3}} + a \right| + a = 0$$

$$\frac{1}{\sqrt{3}} + a = -a$$

$$\frac{1}{\sqrt{3}} + a = -a \rightarrow a = -\frac{1}{\sqrt{3}}$$

$$x > \frac{1}{\sqrt{3}} \rightarrow -x < -\frac{1}{\sqrt{3}}$$

$$\rightarrow -x < -0/\dots \rightarrow [-x] = -1$$

$$\left[-\frac{1}{\sqrt{3}} \right] = -1$$

$$\lim_{x \rightarrow \frac{1}{4}} \frac{14x - \left[-\frac{9}{x^2} \right]}{24x + \left[\frac{9}{x^2} \right]} = \lim_{x \rightarrow \frac{1}{4}} \frac{14x + 9}{24x + 12} = \frac{-1 + 9}{(-12) + 12} = \frac{1}{0^+} = +\infty$$

$$x > -\frac{1}{4} \rightarrow x^2 < \frac{1}{4} \rightarrow \frac{1}{x^2} > 4 \begin{cases} \frac{9}{x^2} < 1 \rightarrow \left[-\frac{9}{x^2} \right] = -9 \\ \frac{9}{x^2} > 12 \rightarrow \left[\frac{9}{x^2} \right] = 12 \end{cases}$$

$$\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^3 - [x^2]} = \lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x^3 - 1} = \frac{0}{0} \xrightarrow{\text{ل'Hôpital}} \lim_{x \rightarrow 2^+} \frac{2x}{3x^2 - 2x} = \lim_{x \rightarrow 2^+} \frac{2}{x^2 + 4 + 2x} = \frac{2}{16}$$

$$x > 2 \rightarrow x^2 > 4$$

$$\rightarrow [x^2] = 4$$

$$\lim_{x \rightarrow -1} \frac{x^2 + 10x + 14}{12 + 4\sqrt{x}} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow -1} \frac{2x + 10}{\frac{4}{2\sqrt{x}}} = \lim_{x \rightarrow -1} \frac{(2x+10)(\sqrt{x})}{2} = \frac{(-4)(2)}{2} = -4$$

$$\xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow -1} \frac{x^2 + 10x + 14}{12 + 4\sqrt{x}} \times \frac{f + \sqrt{x} - \sqrt{x}}{f + \sqrt{x} - \sqrt{x}} = \lim_{x \rightarrow -1} \frac{(x+1)(x+2)(1)}{4(1+x)} = \lim_{x \rightarrow -1} \frac{(x+2) \times 1}{4} = -\frac{1}{4}$$

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{1+x^2} - \sqrt{1-x}}{\sqrt{1-\cos x}} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0^-} \frac{\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1-x}}}{\frac{-\sin x}{2\sqrt{1-\cos x}}} \times \frac{\sqrt{1+\cos x}}{\sqrt{1+\cos x}} \times \frac{\sqrt{1+x^2} + \sqrt{1-x}}{\sqrt{1+x^2} + \sqrt{1-x}}$$

$$= \lim_{x \rightarrow 0^-} \frac{(1+x^2 - 1 + x)(\sqrt{1+x^2})}{\underbrace{\sqrt{1-\cos x}}_{\sin \frac{x}{2}} \times 2\sqrt{1-x}} = \lim_{x \rightarrow 0^-} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 0^-} \frac{1}{-\sin x} = -1$$

برای سادگی

$$\lim_{x \rightarrow 0} \frac{k + \cos(\sqrt{x})}{kx^2} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{-1 + \cos \sqrt{x}}{-2x} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 0} \frac{-\frac{1}{2\sqrt{x}}}{-2} = \frac{1}{4}$$

کسر منفرجه بود

$k + \cos \sqrt{x} = 0 \rightarrow k = -1$

$\cos 0 = 1$

$\lim_{x \rightarrow 0} \frac{1}{\sqrt{x}} = \infty$

$$\lim_{x \rightarrow 9a^+} \frac{\sqrt{x-9a} + \sqrt{x} - \sqrt{9a}}{\sqrt{x-9a} \times \sqrt{x+9a}} = \frac{0}{0} \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow 9a^+} \frac{\frac{1}{2\sqrt{x-9a}} + \frac{1}{2\sqrt{x}}}{\frac{x}{\sqrt{x-9a} \times \sqrt{x+9a}}} = \frac{\frac{1}{2\sqrt{9a}} + \frac{1}{2\sqrt{9a}}}{\frac{9a}{\sqrt{9a} \times \sqrt{9a}}} = \frac{\frac{1}{\sqrt{9a}}}{\frac{9a}{9a}} = \frac{1}{\sqrt{9a}}$$

$$\lim_{x \rightarrow -1} \frac{1-k[x]}{x^2-1} = -\infty \xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow -1^+} \frac{1-k[x]}{x^2-1} = \frac{1+k}{0^-} = -\infty \rightarrow 1+k > 0 \rightarrow k > -1$$

$$\xrightarrow{\text{L'Hôpital}} \lim_{x \rightarrow -1^-} \frac{1-k[x]}{x^2-1} = \frac{1+k}{0^+} = -\infty \rightarrow 1+k < 0 \rightarrow k < -1$$

$$\left. \begin{aligned} -1 < k < -\frac{1}{2} &\rightarrow x < k < -\frac{x}{2} \\ -1 < \cos k < 0 &\rightarrow \end{aligned} \right\} \rightarrow \text{طول و عرض نقطه داده شود}$$

ناحیه سوم است