دوازدهم ۲ پسر A — تکلیف شماره ۲۰ — ایلیا سالاری

1

$$\lim_{x\to 2}\frac{3x-5}{x^2+ax+b}=-\infty \Rightarrow \frac{-1}{4+2a+b}=-\infty \Rightarrow 2a+b+4=0 \Rightarrow 2a+b=-4 \Rightarrow$$

$$x^2+ax+b=\left(x+\frac{a}{2}\right)^2=(x-2)^2=x^2-4x+4 \Rightarrow a=-4,\ b=4 \qquad a+b=(-4)+4=0$$

2

$$\lim_{x\to \sqrt[3]{4}}\frac{x}{x^2+ax+b}=+\infty \Rightarrow \frac{\sqrt[3]{4}}{\sqrt[3]{16}+\sqrt[3]{4}a+b}=-\infty \Rightarrow \sqrt[3]{4}a+b+\sqrt[3]{16}=0 \Rightarrow \sqrt[3]{4}a+b=-\sqrt[3]{16}$$

$$x^2+ax+b=\left(x+\frac{a}{2}\right)^2=(x-\sqrt[3]{4})^2=x^2-2\sqrt[3]{4}x+\sqrt[3]{16} \Rightarrow a=-2\sqrt[3]{4},\ b=\sqrt[3]{16} \qquad \left[\frac{b}{a}\right]=\left[\frac{2\sqrt[3]{2}}{-2\sqrt[3]{4}}\right]=\left[\frac{-\sqrt[3]{4}}{2}\right]=-1$$

3

$$\lim_{x\to\left(\frac{1}{\sqrt{3}}\right)^+}\frac{[x]+a}{\left|\frac{\sqrt{3}}{3}x+a\right|+a}=-\infty \Rightarrow \frac{a-1}{\left|\frac{1}{3}+a\right|+a}=-\infty \Rightarrow \left|a+\frac{1}{3}\right|+a=0 \Rightarrow \begin{cases} a\geq \frac{1}{3} \Rightarrow 2a+\frac{1}{3}=0 \Rightarrow \dfrac{-\frac{7}{6}}{0^-}=-\infty \Rightarrow \\ a<\frac{1}{3} \Rightarrow -a-\frac{1}{3}-a=0 \Rightarrow -\frac{1}{3}=0 \quad \text{غقق} \end{cases}$$

$$\Rightarrow a=-\frac{1}{6}$$

4

$$\lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{16x-\left[-\frac{2}{x^2}\right]}{24x+\left[\frac{3}{x^2}\right]} \Rightarrow \lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{16x+9}{24x+12}=\frac{1}{0^+}=+\infty$$

5

$$\lim_{x\to 2^+}\frac{x^2-4}{x^3-[x^3]}=\frac{x^2-4}{x^3-8}=\frac{(x-2)(x+2)}{(x-2)(x^2+2x+4)}=\frac{x+2}{x^2+2x+4}=\frac{4}{12}=\frac{1}{3}$$

6

$$\lim_{x\to -8}\frac{x^2+10x+16}{12+6\sqrt[3]{x}}=\frac{(x+8)(x+2)}{6(2+\sqrt[3]{x})}=\frac{(\sqrt[3]{x}+2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)(x+2)}{6(2+\sqrt[3]{x})}=\frac{(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)(x+2)}{6}=\frac{4\times(-6)}{6}=-4$$

7

$$\lim_{x\to 0^-}\frac{\sqrt{2+3x}-\sqrt{2-x}}{\sqrt{1-\cos x}}\times\frac{\sqrt{2+3x}+\sqrt{2+x}}{\sqrt{2+3x}+\sqrt{2+x}}=\frac{2+3x-2+x}{\sqrt{\frac{x^2}{2}}\times 2\sqrt{2}}=\frac{4x}{|x|\times 2}=-2 \qquad 1-\cos x\sim\frac{x^2}{2}$$

8

$$\lim_{x\to 0}\frac{k+\cos(\sqrt{a}x)}{kx^2}=3 \Rightarrow \frac{k+\cos 0}{0^-}=3 \Rightarrow k+1=0 \Rightarrow k=-1 \Rightarrow \lim_{x\to 0}\frac{\cos(\sqrt{a}x)-1}{-x^2}\overset{\text{هوپیتال}}{\Rightarrow}$$

$$\lim\frac{-\sqrt{a}\sin\sqrt{a}x}{-2x}=\frac{0}{0}\overset{\text{هوپیتال}}{\Rightarrow}\frac{-a\cos\sqrt{a}x}{-2}=\frac{-a}{-2}=\frac{a}{2}=3 \Rightarrow a=6 \qquad \frac{a}{k}=\frac{6}{-1}=-6$$

9

$$\lim_{x\to(3a)^+}\frac{2\sqrt{x-3a}+3\sqrt{x}-\sqrt{27a}}{\sqrt{x^2-9a^2}}=\frac{0}{0}\Rightarrow \lim\frac{2}{\sqrt{x+3a}}+\frac{3\sqrt{x}-3\sqrt{3a}}{\sqrt{x^2-9a^2}}=\frac{2}{\sqrt{6a}}+\frac{3(x-3a)}{\sqrt{x+3a}\times\sqrt{x-3a}\times 2\sqrt{3a}}=$$

$$\frac{2}{\sqrt{6a}}+0=\frac{2}{\sqrt{6a}}$$

10

$$\lim_{x\to -1}\frac{1-k[x]}{x^2-1}$$

$$(+)\quad \frac{1+k}{x^2-1}=\frac{1+k}{0^-}=-\infty \Rightarrow 1+k>0 \Rightarrow k>-1$$

$$(-)\quad \frac{1+2k}{x^2-1}=\frac{1+2k}{0^+}=-\infty \Rightarrow 1+2k<0 \Rightarrow k<-\frac{1}{2}$$

$$-1<k<-\frac{1}{2} \Rightarrow -\pi<k\pi<-\frac{\pi}{2}=x<0$$

$$-1<\cos k\pi<0 \Rightarrow y<0$$

ربع چهارم