

$$\Delta = 0 \rightarrow (m+3)^2 - 12m = 0 \rightarrow (m-3)^2 = 0 \rightarrow m=3, \quad \lambda = a = \frac{-(m+3)}{2(4)} \xrightarrow{m=3} \lambda = a = -\frac{1}{2}$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{\sqrt{4(x+\frac{1}{2})^2}}{|2x^3+\frac{1}{2}|} = \lim_{x \rightarrow \frac{1}{2}} \frac{\sqrt{4}|x+\frac{1}{2}|}{2|x^3+\frac{1}{2}|} = \lim_{x \rightarrow \frac{1}{2}} \frac{\sqrt{4}|x+\frac{1}{2}|}{2|x+\frac{1}{2}||x^2-\frac{x}{2}+\frac{1}{2}|} = \frac{\sqrt{4}}{\frac{3}{2}} = \frac{2\sqrt{4}}{3} \rightarrow f(-\frac{1}{2}) = \frac{2 \tan b}{\sqrt{1-\frac{1}{4}}} = \frac{2\sqrt{4}}{3} \rightarrow \tan b = \frac{\sqrt{3}}{3} \rightarrow \boxed{b = \frac{\pi}{4}}$$

$$\begin{aligned} x^2 + ax + b = 0 \xrightarrow{x=1} 1 + a + b = 0 \\ 5x^2 - ax + b = 0 \xrightarrow{x=1} 5 - a + b = 0 \Rightarrow \begin{cases} b = -3 \\ a = 2 \end{cases} \end{aligned}$$

در $x=1$ تابع در خارج ناحیه است
چون $x=1$ در ناحیه صورت نیز هست

$$\left[\frac{b-1a}{3} \right] = \left[\frac{-3-2}{3} \right] = -\frac{5}{3}$$

$$x=1 \begin{cases} f(1) = \tan \frac{3\pi}{4} = -1 \Rightarrow -\frac{3}{a} = -1 \Rightarrow a=3 \\ \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{|x^2+x-1|}{a(1-x)} = \lim_{x \rightarrow 1^+} \frac{(x+2)(x-1)}{a(1-x)} = -\frac{3}{a} \end{cases}$$

در $[1, 5]$ یکنواخت است
چون $x=1$ یکنواختی راست
در $x=5$ یکنواختی چپ دارد

$$x=5 \begin{cases} f(5) = b(5-[-5]) = 10b \\ \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^-} \frac{|x^2+x-1|}{3(1-x)} = \frac{11}{-12} = -\frac{11}{12} \Rightarrow 10b = -\frac{11}{12} \Rightarrow b = -\frac{11}{120} \\ \Rightarrow ab = -\frac{11}{40} \end{cases}$$

$$f(x) = a[x+1] + b[x+[a+1]] = a([x]+1) + b([x]+[a]+1)$$

$$f \text{ یکنواخت است بنابراین ضرب } [x] \text{ ضرایب}$$

$$\rightarrow (a+b)[x] + (a+b+b[a]) \rightarrow a+b=0 \rightarrow b=-a \rightarrow f(x) = -a[a] \rightarrow \frac{a[a]}{f(a)} = \frac{a[a]}{-a[a]} = -1$$

$$f(x) = -2a \leftarrow b=0 \text{ که شامل جزء صحیح نباشد بنابراین}$$

$$\rightarrow \frac{a}{f(b)} = \frac{a}{f(0)} = \frac{a}{-2a} = -\frac{1}{2}$$

$f(a) = 0 \leftarrow \lambda = 2a$ ریشه صورت است، $\lambda = a$ ریشه خارج است

$$f(x) = \frac{(x-a)(x-2a)}{-(x-a)} = -(x-2a) \rightarrow f(x) = \begin{cases} -(x-2a) & x \neq a \\ 2 & x = a \end{cases}$$

$$f(a) = -(a-2a) = a = 2 \quad f(x) = \frac{(x-a)(x-2a)}{-(x-a)} = \frac{(x-1)(x-2)}{-(x-1)} = \frac{x^2 - 4x + 2}{-(x-1)}$$

$$= \frac{x^2 + mx + n}{-(x-1)} \Rightarrow m = -4, n = 2 \Rightarrow n - m = 2 - (-4) = 6$$

$$n \in \mathbb{Z} \quad (1-a)n + (2a^2-1)(-n) = b \sin\left(\frac{\pi}{a}\right) \rightarrow n(2-a-2a^2) = b \sin\left(\frac{\pi}{a}\right)$$

$$n \in \mathbb{Z} \quad 2-a-2a^2 = 0 \Rightarrow a = \frac{1}{2}, a = -1$$

$$a = \frac{1}{2} \rightarrow \sin\left(\frac{\pi}{\frac{1}{2}}\right) = -1 \Rightarrow b = 0 \quad \Rightarrow \frac{a}{b} = \frac{\frac{1}{2}}{0} = \infty$$

$$a = -1 \rightarrow \sin(-\pi) = 0 \Rightarrow b \in \mathbb{R}$$

$$\lim_{x \rightarrow 1} f(x) = f(1) = \frac{m}{m+2}$$

$$\lambda = 1 \xrightarrow{\text{صورت}} (x^2 + mx - m - 1) = (x-1)(x^2 + m+1)$$

$$\xrightarrow{\text{خارج}} m|x-1| + |x-1| = |x-1|(m+1)$$

$$\Rightarrow f(x) = \frac{|x-1|(m+1)}{|x-1|(2m+1)} = \frac{m+1}{2m+1}$$

$$\text{نقطه سرجی: } \frac{m+1}{2m+1} = \frac{m}{m+2} \rightarrow \frac{m+1}{m+2} = \frac{m}{m+2} \rightarrow m = 1, m = -1$$

$$\rightarrow \lim_{x \rightarrow -1} f(x) = \frac{m+1}{2m+1} = \lim_{x \rightarrow -1} f(x) \stackrel{m=1}{\frac{m=-1}{-1}} = \frac{1}{-1} = -1 \quad \frac{0}{0} = \frac{1}{2}$$

$$\sqrt{2} |a^2 + a| = 0 \Rightarrow a = -1, a = 0$$

$\lambda = a$ ریشه صورت و خارج یکسان است:

$$f(x) = \frac{\sqrt{2} |x+1|}{|x^2 + (m-2)x + 1|}$$

$$\lambda = -1 \rightarrow |(-1)^2 - (m-2) + 1| = 0 \Rightarrow m = 2$$

$$f(x) = \frac{\sqrt{2} |x+1|}{|x^2 + 1|}$$

$$\lim_{x \rightarrow -1} \frac{\sqrt{2} |x+1|}{|x^2 + 1|} = \lim_{x \rightarrow -1} \frac{\sqrt{2} |x+1|}{|x+1||x^2 - x + 1|} = \lim_{x \rightarrow -1} \frac{\sqrt{2}}{|x^2 - x + 1|} = \frac{\sqrt{2}}{2}$$

$$\lim_{x \rightarrow 0^+} \frac{x^2 + |x|}{x^2 + a|x|} = \frac{0}{0} = \lim_{x \rightarrow 0^+} \frac{x^2 + x}{x^2 + ax} = \lim_{x \rightarrow 0^+} \frac{x+1}{x+a} = \frac{1}{a}$$

$$\lim_{x \rightarrow 0^-} \frac{x^2 + |x|}{x^2 + a|x|} = \frac{0}{0} = \lim_{x \rightarrow 0^-} \frac{x^2 - x}{x^2 - ax} = \lim_{x \rightarrow 0^-} \frac{x-1}{x-a} = \frac{-1}{a}$$

$$f(0) = \frac{a-1}{2a+1} = \frac{1}{a} \Rightarrow 2a^2 - a = 2a+1 \Rightarrow 2a^2 - 2a - 1 = 0 \Rightarrow a = \frac{1}{2}, -\frac{1}{2} \Rightarrow \text{سویکت نیست}$$

$$a = 2 \rightarrow \lim_{x \rightarrow \frac{1}{2}} f(x) = \lim_{x \rightarrow \frac{1}{2}} \frac{x^2 + x}{x^2 + 2x} = \frac{\frac{1}{4} + \frac{1}{2}}{\frac{1}{4} + 1} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$