۱- حسن شهبازی

۲-

$5x^2 - ax + b = 0 \quad 5 - a + b = 0$

$b - a = -5 \quad b = -3 \qquad \left[\frac{\overset{-4}{-3-2(2)}}{3}\right] = \left[-\frac{7}{3}\right] = -3$

$b + a = -1 \quad a = 2$

$f'(x) =$

$f(1) \quad \lim_{x\to 1} f(x) \Rightarrow \lim_{x\to 1}\frac{x^2+ax+b}{x-1} = \frac{0}{0} \xrightarrow{\text{ریشه}} x^2+ax+b=0 \quad 1+a+b=0 \quad a+b=-1$

۳-

$$f(x) = \begin{cases} \tan\dfrac{(2x+1)\pi}{4} & ; x<1 \\ \dfrac{|x^2+x-2|}{a(1-x)} & ; 1<x<5 \\ b(x-[-x]) & ; x\geqslant 5 \end{cases}$$

پیوسته $[1, 5]$

$\lim_{x\to 1^+} f(1) = f(1) \quad , \quad \lim_{x\to 5^-} f(5) = f(5)$

$\lim_{x\to 1^+}\frac{|x^2+x-2|}{a(1-x)} = \frac{0}{0} \xrightarrow[\text{اصلی}]{\text{رفع}} \frac{\cancel{(x-1)}(x+2)}{-a\cancel{(x-1)}} = \frac{3}{-a}$

$f(1) = \tan\frac{3\pi}{4} = -1 \qquad \frac{3}{-a} = -1 \quad \boxed{a=3}$

$\lim_{x\to 5}\frac{|x^2+x-2|}{3(1-x)} = \frac{28}{3\times -4} = -\frac{7}{3} \qquad \boxed{ab = 3\times -\frac{7}{30} = -\frac{7}{10}}$

$f(5) = b(\cancel{5-[-5]}^{10}) = 10b = -\frac{7}{3} \quad \boxed{b = -\frac{7}{30}}$

$f(x) = a[x+1] + b[x + [a+1]] \longrightarrow a[x+1] + b[x] + b[a+1]$

$a[a+1] + b[a] + b[a+1] = a(a+1) + b(a) + b\cancel{a}a + b = a^2 + 2ab + a + b^{a+b} \longrightarrow a^2 + 2a(-a) \cancel{+a} - a$

$a^2 - 2a^2 = -a$

$\lim_{x \to a^-} f(x) = \lim a[a^- + 1] + b[a^-] + b[a+1] = a(a) + b(a-1) + b(a+1)$

$[a^-] = a - 1$ $\qquad a^2 + ba - b + ba + b = a^2 + 2ba$

$a^2 + 2ba + a + b = a^2 + 2ba \Longrightarrow a = -b \quad \frac{\cancel{a}[a]}{-\cancel{a}} = -[a] = -a$

۵-

$f(x) = b[x^2 - ax] - 2a \qquad D_f = \mathbb{R}$

زمانی ممکن است که $\boxed{b = 0}$ تابع ثابت $f(x) = -2a$

$\frac{a}{f(b)} \Rightarrow \frac{a}{-2a} = \frac{1}{-2}$

۶-

$$f(x) = \begin{cases} \frac{x^2 + mx + n}{a - x} & ; x \neq a \\ 2 & ; x = a \end{cases} \qquad f(2a) = 0$$

$\lim_{x \to a} \frac{x^2 + mx + n}{a - x} = 2 \Longrightarrow \lim_{x \to a} \frac{x^2 - 3ax + 2a^2}{a - x} = 2$

$x^2 + mx + n \longrightarrow (a - x)(2a - x) = x^2 - 3ax + 2a^2$

$\lim_{x \to a} \frac{x^2 - 3ax + 2a^2}{a - x} = \frac{0}{0} \xrightarrow{\text{رفع ابهام}} \frac{\cancel{(a-x)}(2a-x)}{\cancel{a-x}} = 2a - x$

$\lim_{x \to a} 2a - x = 2 \qquad 2a - a = 2 \quad \boxed{a = 2}$

$m + n \Longrightarrow -3a + 2a^2 \Longrightarrow -3(2) + 2(2)^2 = -6 + 8 = 2$

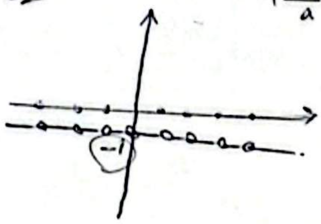
-7

$$f(x)=\begin{cases}(1-a)[x]+(3a^2-1)[-x] & x\notin Z\\ b\sin\left(\frac{\pi}{a}\right) & x\in Z\end{cases}$$

$3a^2-1=1-a \quad 3a^2+a-2=0 \quad (a+1)(a-\frac{2}{3})=0 \to -1,\ \frac{2}{3}$

$b\sin\left(\frac{\pi}{a}\right)=-1 \quad \sin(-\pi)=0$ غ ق ق

$\sin\left(\frac{3\pi}{2}\right)=-1 \quad a/b=\frac{2/3}{1}=\frac{2}{3}$



$-b=-1 \quad b=1 \quad a=\frac{2}{3}$

-8

$$f(x)=\begin{cases}\dfrac{|x^2+mx-m-1|}{m|x^2-1|+|x-1|} & ;\ x\neq 1\\[2ex] \dfrac{m}{m+2} & ;\ x=1\end{cases}$$

$\lim_{x\to 1}\dfrac{|x^2+mx-m-1|}{m|x^2-1|+|x-1|} \to \dfrac{\cancel{(x-1)}(x+1+m)}{\cancel{(x-1)}(m(x+1)+1)}=\dfrac{2+m}{2m+1}$

$\to \dfrac{-\cancel{(x-1)}((x+1)+m)}{\cancel{(x-1)}(-m(x+1)-1)}=\dfrac{-(2+m)}{-(2m+1)}$

$\dfrac{m}{m+2}=\dfrac{2+m}{2m+1} \quad 2m^2+m=m^2+4m+4$

$m^2-3m-4=(m-4)(m+1)=0 \to m=-1,\ m=4$

$\lim_{x\to -1}\dfrac{|x^2-x|}{-|x^2-1|+|x-1|}=\dfrac{2}{2}=1$

$f(x)=\dfrac{\sqrt{3}\,|ax+a|}{|x^3+(m-2)x+a^2|} \qquad R-\{a\}$

$x^3+(m-2)x+a^2=0 \quad \{\text{if } a\neq 0 \to a^3+(m-2)a+a^2=0 \quad m=-a^2-a+2$

$x^3+(-a^2-a)x+a^2=x^3-a^2x-ax+a^2=(x-a)(x^2+ax-a)$

$\lim_{x\to a}\dfrac{\sqrt{3}\,|a(x+1)|}{|(x-a)(x^2+ax-a)|}=\dfrac{\sqrt{3}\,|a^2+a|}{0}=+\infty$

۱۰ـ

$$f=\begin{cases}\dfrac{x^2+|x|}{x^2+a|x|} & x\neq 0\\[2ex] \dfrac{2a-1}{2a+2} & x=0\end{cases}\qquad \lim_{x\to 0}\frac{x^2+|x|}{x^2+a|x|}=f(0)\Rightarrow \lim_{x\to 0}\frac{2x+1}{2x+a}=\frac{1}{a}$$

$$\frac{1}{a}=\frac{2a-1}{2a+2}\Rightarrow 2a^2-a=2a+2$$

$$2a^2-3a+2=0\quad (a-2)(a+\tfrac{1}{2})=0 \begin{cases}2 & \text{ق ق}\\ -\tfrac{1}{2} & \text{غ ق ق}\end{cases}$$

$$\lim_{x\to\frac{1}{2}}f(x)=\frac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{4}+2\underbrace{(\cancel{\tfrac{1}{2}})}_{1}}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{3}{5}$$

$$\lim_{x\to -2}f(x)=\frac{4+2}{4-\underbrace{\cancel{\tfrac{|-2|}{2}}}_{|-1|}}=\frac{6}{3}=2$$