تکلیف شماره ۲۱ — کلاس ۱۲ ریاضی A — میررضا ابراهیمی — ۲۰

۱-

$\lim_{x\to a} f(x) = f(a) \overset{\text{پیوسته}}{\Rightarrow} |2a^3 + ma^2 - 3a^2 + a^2| = 0 \Rightarrow 2a^3 + ma^2 - 2a^2 = 0 \Rightarrow a^2(2a+m-2)=0$

$\Rightarrow 2a + m - 2 = 0 \Rightarrow m = 2-2a \Rightarrow 4a^2 + (m+3)a + \frac{m}{2} = 0$

$\Rightarrow 4a^2 + (2-2a+3)a + \frac{2-2a}{2} = 0 \Rightarrow 4a^2 + 5a - 2a^2 + 1 - a = 4a^2 + 4a + 1 = 0$

$(2a+1)^2 = 0 \Rightarrow 2a = -1 \Rightarrow a = -\frac{1}{2} \Rightarrow m = 3$

صورت $\Rightarrow \sqrt{4x^2 + 4x + \frac{3}{2}} = \sqrt{\frac{3}{2}(4x^2+4x+1)} = \sqrt{\frac{3}{2}}\,|2x+1|$ ۲

مخرج $\Rightarrow |2x^3 + \frac{1}{4}| = |\frac{1}{4}(8x^3+1)| = |\frac{1}{4}(2x+1)(4x^2+1-2x)|$

$\Rightarrow \lim_{x\to a} \frac{\sqrt{1/5}\,\cancel{|2x+1|}}{\frac{1}{4}\cancel{|2x+1|}\,|4x^2+1-2x|} = \frac{\sqrt{1/5}}{\frac{1}{4}(\frac{4}{4}+1+1)} = \frac{\sqrt{\frac{3}{2}}}{\frac{3}{4}} = \frac{\frac{\sqrt{6}}{2}}{\frac{3}{4}} = \frac{2\sqrt{6}}{3}$

$\frac{2\tan b}{\sqrt{\frac{1}{2}}} = 2\sqrt{2}\tan b = \frac{2\sqrt{6}}{3} = \tan b = \frac{\sqrt{3}}{3} \Rightarrow b = \frac{\pi}{6}$ ✓

۲-

$\lim_{x\to 1} \frac{x^2+ax+b}{x-1} \overset{0/0}{\Rightarrow} x^2+ax+b=0 \quad 5x^2 - ax + b = 0 \Rightarrow b-a=-5$

$b+a=-1 \Big\} b=-3 \Rightarrow a=2$

$\Rightarrow [\frac{-7}{3}] = -3$ ✓ ۲

۳-

$\tan\frac{3\pi}{4} = -1 \Rightarrow \lim_{x\to 1^+} \frac{(x+2)(x-1)}{a(1-x)} = -\frac{3}{a} = -1 = a = 3$ ۲

$\lim_{x\to 5} \frac{|x^2+x-2|}{3(1-x)} = -\frac{7}{3} \Rightarrow f(5) = b(\frac{x}{5}+5) = -\frac{7}{3} \Rightarrow b = -\frac{7}{30}$

$= 10b = -\frac{7}{10} = -0/7$ ✓

۴-

$a[x]+a + b[x] + b[a] + b \Rightarrow b = -a \Rightarrow a + -a[a] - a \Rightarrow f(x) = -a[a]$

$\Rightarrow \frac{a[a]}{f(a)} = -1$ ✓ ۲

۵-

$b=0 \Rightarrow f(x) = -2a \Rightarrow \frac{a}{-2a} = -\frac{1}{2}$ ✓ ۲

۶-

$a^2+am+n=0 ,\ \frac{4a^2+2am+n}{-a} = 0 \Rightarrow a^2+am+n = 4a^2+2am+n$

$\Rightarrow 3a^2+am=0 \Rightarrow a(3a+m)=0 \begin{cases} a\neq 0 \\ m=-3a \end{cases} \Rightarrow a^2-3a^2+n=0$

$\Rightarrow n = 2a^2 ,$

$\lim_{x\to a} \frac{x^2-3ax+2a^2}{a-x} = 2 \Rightarrow \frac{\cancel{(x-a)}(x-2a)}{\cancel{a-x}_{-1}} = 2 \Rightarrow a = 2 \Rightarrow n = 8 \Rightarrow m = -6$

$n - m = 14$ ✓ ۲

۷-

$[-x] = -1 - [x] \Rightarrow f(x) = (1-a)[x] + (3a^2-1)(\underbrace{[-x]}_{-1-[x]})$

$\Rightarrow f(x) = (1-a)[x] - (3a^2-1) - (3a^2-1)[x]$

$\Rightarrow f(x) = ((1-a)-(3a^2-1))[x] - (3a^2-1) \Rightarrow f(x) = (2-a-3a^2)[x] - 3a^2-1$

فرض پرسش

$\Rightarrow 2-a-3a^2 = 0 \Rightarrow a = -1 \,,\, \frac{2}{3} \Rightarrow 1-3a^2 = f(x)$

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$\Rightarrow 1-3a^2 = b\sin(\frac{\pi}{a}) \Rightarrow a \neq -1 \Rightarrow a = \frac{2}{3} \Rightarrow -\frac{1}{3} = -b \Rightarrow b = \frac{1}{3}$

$\Rightarrow \frac{a}{b} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$ ✓

۸-

$\lim_{x\to 1} \frac{|(x+m+1)\cancel{(x-1)}|}{\cancel{(x-1)}(m|x+1|+1)} \Rightarrow \frac{2+m}{|2m+1|} = \frac{m}{m+2} \Rightarrow m^2-3m-4=0 \Rightarrow m=4 \,,\, m=-1$

if $m=-1 \Rightarrow 1=-1$ ✗ $\Rightarrow m=4 \Rightarrow f(x) = \frac{|x+m+1|}{m|x+1|+1} \Rightarrow f(\frac{1}{4}) = \frac{|\frac{1}{4}+5|}{6}$

$\Rightarrow \frac{\frac{21}{4}}{6} = \frac{21}{24} = \frac{7}{8}$ ✓

2

۹-

$\lim_{x\to a} \frac{\sqrt{3}\,|ax+a|}{|x^3+(m-2)x+a^2|} \overset{\frac{0}{0}}{\Rightarrow} \sqrt{3}\,|a(a)+a| = 0 \Rightarrow (a^2+a) = 0$

$a=0$ ✗ , $a=-1$ ✓ $\Rightarrow (-1)^3 + (m-2)(-1) + (-1)^2 = 0 \Rightarrow m-2=0 \Rightarrow m=2$

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$\Rightarrow \lim_{x\to -1} \frac{\sqrt{3}\,|-x-1|}{|x^3+1|} \Rightarrow \frac{\sqrt{3}\,\cancel{|x+1|}}{\cancel{|x+1|}\times|x^2-x+1|} = \frac{\sqrt{3}}{|x^2-x+1|} \Rightarrow \frac{\sqrt{3}}{|1+1+1|} = \frac{\sqrt{3}}{3}$ ✓

$x=0 \begin{cases} + \quad \frac{x^2+x}{x^2+ax} = \frac{x+1}{x+a} = \frac{1}{a} \\ - \quad \frac{x^2-x}{x^2-ax} = \frac{x-1}{x-a} = \frac{1}{a} \end{cases} \Rightarrow \frac{1}{a} = \frac{2a-1}{2a+2} \Rightarrow 2a^2-3a-2=0$

$\Rightarrow a = -\frac{1}{2}$ ✗ , 2 ✓ $\Rightarrow f(\frac{1}{2}) = \frac{\frac{1}{4}+\frac{1}{2}}{\frac{1}{4}+1} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$ ✓

مخالف
[illegible]

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