نام و نام خانوادگی ......محمدیار همت کار...... ~~۱۷~~ پاسخنامه تشریحی تکلیف شماره .... کلاس ....

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$$f(x) = \begin{cases} a+|x| & x<1 \\ b\sqrt[3]{x^2} & x \geq 1 \end{cases}$$

تابع فقط در $x=0$ مشتق ناپذیر است (ریشه‌ی سادهٔ قدرمطلق)

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$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow a+1 = b \Rightarrow a-b=-1$$

$$f'_-(1) = f'_+(1) \Rightarrow 1 = b \times \frac{2}{3} \times 1 \Rightarrow b = \frac{3}{2}$$

$\Big\}\ a = \frac{1}{2}$ ✓ $a+b = 2$

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$$2g(x) = \frac{2|x-2| + 2\sqrt{3x}}{\sqrt[3]{x^2}} \Rightarrow (f-2g)(x) = \frac{-2\sqrt{3x}}{\sqrt[3]{x^2}} = -2\sqrt{3} \times x^{-\frac{1}{6}}$$

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$$(f-2g)'(x) = 2\sqrt{3} \times \frac{1}{6} \times x^{-\frac{7}{6}} \Rightarrow (f-2g)'(1) = \frac{\sqrt{3}}{3}$$ ✓

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$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a) \Rightarrow \frac{1}{a} = ab+c$$

$$f'_+(a) = f'_-(a) \Rightarrow -\frac{1}{a^2} = b$$

$$\Bigg\}\ \frac{1}{a} = a\left(-\frac{1}{a^2}\right) + c$$

$$\frac{2}{a} = c \Rightarrow ac = 2$$ ✓ ۲

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$$f'_-(a) \neq f'_+(a)$$

$0 \neq m(x-a)^{m-1} \Rightarrow$ تنها مقدار صحیح نامنفی m یک است $\Rightarrow$ ۱ مقدار ✓

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$$g'(-\sqrt[3]{2}) \cdot f'(g(-\sqrt[3]{2})) = (fog)'(-\sqrt[3]{2})$$

$$-\sqrt[3]{2} < 0 \Rightarrow |x| = -x,\ |x^3| = -x^3 \Rightarrow fog(x) = x$$ ۲

$$fog'(-\sqrt[3]{2}) = 1$$ ✓

$f'(3) = -a \qquad f(3) = -a(3) - 5 = -3a - 5$ 1,5 $-a + 3a + 5 = 2 \Rightarrow 2a = -3 \Rightarrow a = -\frac{3}{2}$ $\frac{f(3)}{f'(3)} = \frac{19}{3} \qquad \frac{f(3)}{f'(3)} = \frac{-3a-5}{-a} = \frac{-3(-\frac{3}{2})-5}{\frac{3}{2}} = \frac{-1}{3}$	6
$(f\circ g)'(x) = g'(x)\, f'(g(x)) \qquad g(x) = (x^2-1)^{-\frac{1}{3}} \Rightarrow g'(x) = -\frac{1}{3}(2x)(x^2-1)^{-\frac{4}{3}}$ $\Rightarrow g'(\frac{3}{\sqrt{8}}) = -8\sqrt{2} \qquad g(\frac{3}{\sqrt{8}}) = 2$ 2 $f'(2) = 2(4x)\times 4 = 64$ $\left.\right\} (f\circ g)'(\frac{3}{\sqrt{8}}) = -8\sqrt{2}\times 64$ ۴ برابر $-128\sqrt{2}$	7
$T = 5 \Rightarrow f(3x+10) = f(3x+5)$ 2 $g'(-2) = f'(-1) + 3f'(-1) = 4f'(-1) = 4\times\frac{3}{2} = 6$	8
$\lim_{h\to 0}\frac{(f(5-h)-2)(f(5-h)-1)}{h(5-h)} = \lim_{h\to 0}\frac{f(5-h)-f(5)}{h}\times\lim_{h\to 0}\frac{f(5-h)-1}{5-h}$ $-\frac{25}{12}\times\frac{1}{5} = -\frac{5}{12}$ $= -f'(5)\times\frac{1}{5} = -\cancel{\frac{9}{4}\times\frac{1}{5}} = -\frac{9}{20}$ $\qquad f'(x) = \sqrt[3]{x+3} + (x-4)\frac{1}{3\sqrt[3]{(x+3)^2}}$ 1 $\rightarrow f'(5) = 2 + \frac{1}{12} = \frac{25}{12}$ $f'(x) = 1\times\sqrt[3]{x+3} + \frac{1}{3\sqrt[3]{(x+3)^2}}\times(x-4)$	9
$6y - 3x = 1 \longrightarrow$ شیب خط $= \frac{1}{2} \longrightarrow$ شیب خط عاس $= -2$ $f'(x) = \frac{3}{6} = \frac{1}{2} \qquad y' = 2x - 4 = -2 \longrightarrow x = 1,\ y = 2 \longrightarrow$ نقطه $(1,2)$ 0,5 $2x - 4 = \frac{1}{2} \Rightarrow x = \frac{9}{4} \Rightarrow y = \frac{1}{2}\times\frac{9}{4} + \frac{1}{6} = \frac{31}{24} \qquad (\frac{1}{2}, \frac{31}{24})$	10