مبین شیبانی

$$f(x)=\frac{|۲x-۴|}{\sqrt[۳]{x^۲}}\qquad f'(x)=\cancel{[illegible]}\,-۲\left(x^{-\frac{۲}{۳}}\right)+-\tfrac{۲}{۳}x^{-\frac{۵}{۳}}\left(۲x-۴\right)\qquad f'(۱)=-۲+\tfrac{۴}{۳}=-\frac{۲}{۳}$$

$$g(x)=\frac{\sqrt{x^۲-۴x+۴}+\sqrt{۳x}}{\sqrt[۳]{x^۲}}$$

$$g'(x)=(۲-x+\sqrt{۳x})\times-\tfrac{۲}{۳}x^{-\frac{۵}{۳}}+\left(-۱+\frac{۳}{۲\sqrt{۳x}}\right)x^{-\frac{۲}{۳}}$$

$$g'(۱)=(۱+\sqrt{۳})\times-\tfrac{۲}{۳}+\left(-۱+\frac{۳}{۲\sqrt{۳}}\right)۱=-\tfrac{۲}{۳}-\frac{۲\sqrt{۳}}{۳}-۱+\frac{\sqrt{۳}}{۲}$$

$$-\tfrac{۵}{۳}\quad\frac{-۴\sqrt{۳}+۳\sqrt{۳}}{۶}=\frac{-\sqrt{۳}}{۶}\longrightarrow-\tfrac{۵}{۳}-\frac{\sqrt{۳}}{۶}$$

$$-\tfrac{۲}{۳}-۲\left(-\tfrac{۵}{۶}-\frac{\sqrt{۳}}{۶}\right)=-\tfrac{۲}{۳}+\tfrac{۱۰}{۳}+\frac{\sqrt{۳}}{۳}=\frac{۸+\sqrt{۳}}{۳}$$

۳-

$$f(x)=\begin{cases}bx+c & x<a\\ \frac{1}{x} & x\geq a\end{cases}\qquad \lim f_+(a)=\lim f_-(a)\qquad \lim_{x\to a^-}bx+c=\lim_{x\to a^+}\tfrac{1}{x}\qquad ba+c=\tfrac{1}{a}$$

$$f'_+(a)=f'_-(a)\qquad b=\frac{-1}{a^۲}\qquad b=\frac{-1}{a^۲}$$

$$-\tfrac{1}{a^۲}\times a+c=\tfrac{1}{a}\qquad c=\tfrac{۲}{a}$$

$$\boxed{ac\Rightarrow a\times\tfrac{۲}{a}=۲}$$

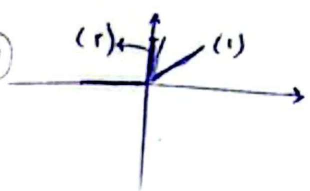
۴-

$$f(x)=\begin{cases}b & ;\ x<a\\ b+(x-a)^m & ;\ x\geq a\end{cases}\qquad f'_-(a)=0$$

$$f'_+(a)=\lim\frac{b+(x-a)^m-b}{x-a}=(x-a)^{m-1}\Rightarrow\lim_{x\to a}(x-a)^{m-1}\neq0$$

$$m=\{0,1\}\qquad \text{if }\lim_{x\to a^+}\frac{(x-a)^m}{x-a},\ m=0\rightarrow\lim_{x\to a}\frac{1}{x-a}=\frac{1}{0^+}=+\infty\qquad 0\leq m\leq1$$

$$\text{if }\lim_{x\to a^+}\frac{(x-a)^m}{x-a},\ m=1\rightarrow\lim_{x\to a^+}\frac{(x-a)}{x-a}=1$$



۵-

$g'(x)f'(g(x)) = f'(g(x)) \longrightarrow$ مشتق این عبارت

$g(x) = \frac{۱}{x^۳-|x^۳|} \longrightarrow x=-\sqrt[۳]{۲} \Rightarrow \frac{۱}{(-\sqrt[۳]{۲})^۳-|(-\sqrt[۳]{۲})^۳|} = \frac{۱}{-۲-۲} = -\frac{۱}{۴}$

$f\circ g = \frac{۱}{\sqrt[۳]{\frac{۱}{x^۳-|x^۳|}-\left|\frac{۱}{x^۳-|x^۳|}\right|}} = \frac{۱}{\sqrt[۳]{\frac{۱}{۲x^۳}-\left|\frac{۱}{۲x^۳}\right|}} = \frac{۱}{\sqrt[۳]{\frac{۱}{۲x^۳}+\frac{۱}{۲x^۳}}} = \frac{۱}{\sqrt[۳]{\frac{۲}{۲x^۳}}} = \frac{۱}{\frac{۱}{x}} = x$

$f\circ g = x$ $\boxed{(f\circ g)' = ۱}$

۶

$f'(۲) = -a \qquad f'(۳)-f(۳) = ۲$

$f(۳) = -۳a-۵ \qquad -a+۳a+۵ = ۲ \qquad ۲a+۵ = ۲ \qquad ۲a = -۳ \qquad a = -\frac{۳}{۲}$

$f(۳) = +\frac{۳}{۲}(۳)-۵ = -\frac{۱}{۲} \qquad \frac{f(۳)}{f'(۳)} = \frac{-\frac{۱}{۲}}{\frac{۳}{۲}} = \boxed{-\frac{۱}{۳}}$

$f'(۳) = -(-\frac{۳}{۲}) = \frac{۳}{۲}$

۷-

$(f\circ g)' = g'(x)f'(g(x)) \longrightarrow \frac{۴}{۳}\times ۱۶ = \frac{۶۴}{۳} \qquad \frac{\cancel{\frac{۶۴}{۳}}}{\frac{-۱۲x\sqrt{۲}}{۲}} = \frac{۱}{-۴\sqrt{۲}} = \frac{-\sqrt{۲}}{۱۲}$

$g(\frac{۳}{\sqrt{۸}}) = \frac{۱}{\sqrt[۳]{\frac{۹}{۸}-\frac{۸}{۸}}} = ۲$

$g'(\frac{۳}{\sqrt{۸}}) = \frac{۳}{۳\sqrt[۳]{\frac{x^۲-۱}{۸}}} = \frac{\frac{۳}{۲}}{\frac{۳}{۲}}... = \frac{۴}{۳}$

$f'(۲) = ۲\times ۲\left[\frac{۴+\frac{۱}{۲}}{۴}\right] = ۱۶$

$T = ۵ \qquad g(x) = f(x+۱)+f(۳x+۱۰) \qquad g'(x) = f'(x+۱)+۳f'(۳x+۱۰)$ ۸-

$f'(-۱) = \frac{۳}{۲} \qquad g'(-۲) = ?$

$g'(-۲) = f'(-۱)+۳f'(۴) \longrightarrow ۴\times\frac{۳}{۲} = ۶$

$۴f'(-۱)$

$f'(-۱) = f'(۴)$

$f(-۱) = f(۴)$

9-

$$f(x) = (x-4)\sqrt[3]{x+3} \qquad f(5) = 2$$

$$\lim_{h \to 0} \frac{f^2(5-h) - 3f(5-h) + 2}{h(5-h)} = \frac{\big(f(5-h)-2\big)\big(f(5-h)-1\big)}{h(5-h)} = \overbrace{\frac{f(5-h)-2}{h}}^{f'(5)} \times \overbrace{\frac{f(5-h)-1}{5-h}}^{f(5)} = \frac{1}{5} \times f'(5)$$

$$f'(x) = \sqrt[3]{x+3} + (x-4)\frac{1}{3\sqrt[3]{(x+3)^2}} \xrightarrow{x=5} 2 + \frac{1}{12} = \frac{25}{12}$$

$$\boxed{\frac{1}{5} \times \frac{25}{12} = \frac{5}{12}}$$

10-

$$6y - 3x = 1 \qquad y_1 = x^2 - 4x + 5 \qquad y_2' = 2x - 4$$

$$y = \frac{3x+1}{6} \qquad m = \frac{1}{2} \qquad 2x - 4 = \frac{1}{2} \qquad 2x = \frac{5}{2} \qquad \boxed{x = \frac{5}{4}}$$