با سلام و ... صادره ریاضی سری A تعلق ۲۳

۱. $f(x)=\begin{cases} a+\sqrt{x^2} & x<1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases} \Rightarrow f(x)=\begin{cases} a+|x| & x<1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases}$

$a+1=b \ ,\ a-b=-1 \Rightarrow b=\frac{3}{2} \ ,\ a=\frac{1}{2} \Rightarrow f'(x)=\begin{cases} -1 & x\leq 0 \\ 1 & 0<x\leq 1 \\ \frac{2b}{3\sqrt[3]{x}} & x\geq 1\end{cases}$

$f(x)=\begin{cases} a-x & x\leq 0 \\ a+x & 0<x\leq 1 \\ b\sqrt[3]{x^2} & x\geq 1\end{cases} \Rightarrow f'(1)=\frac{2b}{3} \Rightarrow b=\frac{3}{2}$

$\Rightarrow a+b=2$

۲. $f(x)=\frac{|3x-4|}{\sqrt[3]{x^2}}=\frac{4-3x}{\sqrt[3]{x^2}} \xrightarrow{\text{مشتق}} f'(x)=\frac{-3\sqrt[3]{x^2}-(4-3x)\frac{2x}{3\sqrt[3]{x^4}}}{\dots}$

$\Rightarrow x=1 \Rightarrow f'(1)=-\frac{10}{3} \ ,\ g(x)=\frac{(2-x)+\sqrt{3x}}{\sqrt[3]{x^2}} \Rightarrow g'(x)=$

$\frac{\left(-1+\frac{3}{2\sqrt{3x}}\right)\left(\sqrt[3]{x^2}\right)-\left(2-x+\sqrt{3x}\right)\left(\frac{2x}{3\sqrt[3]{x^4}}\right)}{\left(\sqrt[3]{x^2}\right)^2}=\frac{-10-\sqrt{3}}{6}$

$\Rightarrow -\frac{10}{3}+\frac{10+\sqrt{3}}{3}=\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$

۳. $f(x)=\begin{cases} bx+c & x<a \\ \frac{1}{x} & x\geq a\end{cases} \quad x=a \Rightarrow ab+c=\frac{1}{a}$

$\Rightarrow -\frac{1}{x^2}=b \Rightarrow b=-\frac{1}{a^2}$

$\Rightarrow -\frac{1}{a}+c=\frac{1}{a} \ ,\ \frac{2}{a}=c \Rightarrow ac=2$

۴. $f(x)=\begin{cases} b & x<a \\ b+(x-a)^m & x\geq a\end{cases} \Rightarrow f'(x)=\begin{cases} 0 & x<a \\ m(x-a)^{m-1} & x>a\end{cases}$

فقط برای $m=1$ تابع نقطه گوشه دارد

$m-1=0 \Rightarrow m=1$

$g'(-\sqrt[3]{2})\, f'(g(-\sqrt[3]{2})) = (f\circ g)'(-\sqrt[3]{2})$ -۵

$\Rightarrow x = -\sqrt[3]{2} \Rightarrow f(x) = \frac{1}{\sqrt[3]{2x}} \Rightarrow f\circ g(x) = \frac{1}{\sqrt[3]{\frac{2}{2x^3}}} = x$

$\Rightarrow (f\circ g)' = (x)' = 1 \Rightarrow x' = 1$

$f(3) = -3a-5 \quad , f'(3) = -a \Rightarrow f(3)' - f(3) = -a + 3a + 5 - 6$

$= 2 \Rightarrow a = -\frac{3}{2} \Rightarrow f(3) = -\frac{1}{2} \Rightarrow \frac{f(3)}{f(3)'} = \frac{-\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{3}$

$, f'(3) = \frac{3}{2}$

-۷

$g(x) = f(x+1) + f(3x+4) \quad g'(x) = f(x+1)' + 3f(3x+4)'$ -۸

$\Rightarrow g(0)' = f(1)' + 3f(4)' \Rightarrow g(0)' = \frac{3}{2} + \frac{9}{2} = \frac{12}{2} = 6$

$\lim \frac{(f(5-x))^2 - 4f(5-x) + 4}{5x - x^2} \overset{HoP}{=} \frac{2(-f(5-x))f(5-x) + 4f'(5-x)}{5-2x}$

$= \frac{f'(5)}{5} = \frac{-\frac{5}{12}}{5} = -\frac{5}{12}$

$4y = 2x+1 \Rightarrow y = \frac{1}{2}x + \frac{1}{4} \Rightarrow m = \frac{1}{2} \Rightarrow m' = -2$ -۱۰

$\Rightarrow (x^2 - 4x + 5)' = -2 \rightarrow 2x - 4 = -2 \rightarrow x = 1$

$x = 1 \Rightarrow x^2 - 4x + 5 = 2 \Rightarrow (1, 2)$