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ارار زاديا

تالیف شماره 22

تلاش خود را در کم دیگر کرده

$$f(x) \begin{cases} a + \sqrt{x} & ; x < 1 \\ b \sqrt{x} & ; x \geq 1 \end{cases}$$

$$f'(x) \begin{cases} \frac{x}{\sqrt{x}} & ; x < 1 \\ b \frac{1}{\sqrt{x}} & ; x \geq 1 \end{cases}$$

1) نقطه‌ری برای برشی $a + 1 = b$

2) نقطه‌ری برای مشتق‌پذیری $a + 1 = \frac{b}{\sqrt{a}}$ (1)

$$\Rightarrow \frac{1}{\sqrt{a}} = 1 \Rightarrow b = \frac{1}{\sqrt{a}} \quad \left. \begin{aligned} a + 1 &= \frac{1}{\sqrt{a}} \\ a + 1 &= \frac{1}{\sqrt{a}} \end{aligned} \right\} a + b = \frac{1}{\sqrt{a}} \quad (2)$$

1. نقطه $f - 2a$

$$f(x) = \frac{|2a - x|}{\sqrt{x}} \Rightarrow f'(x) = \frac{((\sqrt{x})'(-2) - (2a - x)(\frac{1}{\sqrt{x}}))}{x}$$

$$\Rightarrow f'(1) = \frac{(1)(-2) - (2a - 1)(\frac{1}{\sqrt{1}})}{1} = -2 - \frac{2a - 1}{1} = -2 - 2a + 1 = -2a - 1$$

$$g(x) = \frac{\sqrt{x^2 - 2ax + a^2} + \sqrt{2ax}}{\sqrt{x}} \Rightarrow g'(x) = \frac{(\sqrt{x^2 - 2ax + a^2})' + (\sqrt{2ax})'}{\sqrt{x}}$$

$$\Rightarrow g'(1) = -1 + \frac{\sqrt{1} - \sqrt{2a}}{1} = -1 + \sqrt{1} - \sqrt{2a} = -\sqrt{2a}$$

$$f'(1) - g'(1) = -2a - 1 - (-\sqrt{2a}) = -2a - 1 + \sqrt{2a} = \frac{\sqrt{2a}}{\sqrt{2a}} = \frac{\sqrt{2a}}{\sqrt{2a}}$$

$$f(x) \begin{cases} ba + c & ; x < a \\ \frac{1}{x} & ; x \geq a \end{cases}$$

نقطه‌ری a
مشتق‌پذیری $\Rightarrow ab + c = \frac{1}{a} \Rightarrow a^2b + ac = 1$
 $ac = 1 - a^2b$

$$f(x) = f(a) \Rightarrow b = \frac{1}{a^2} \Rightarrow ba^2 = 1$$

$$g(x) = \frac{\sqrt{x^2 - 2ax + a^2} + \sqrt{2ax}}{\sqrt{x}} = \frac{|x - a| + \sqrt{2ax}}{\sqrt{x}} \xrightarrow{x=1} \frac{(-1 + a) + \sqrt{2a}}{\sqrt{1}}$$

$$g'(x) = \frac{(-1 + \frac{1}{\sqrt{2a}})'(1) - \frac{1}{\sqrt{2a}}(1 + \sqrt{2a})}{1} \Rightarrow g'(1) = (-1 + \frac{1}{\sqrt{2a}}) - \frac{1}{\sqrt{2a}}(1 + \sqrt{2a})$$

$$f(x) \begin{cases} b & ; x < a \\ b + (x-a)^m & ; x \geq a \end{cases} \Rightarrow f'(x) \begin{cases} 0 & ; x < a \\ m(x-a)^{m-1} & ; x \geq a \end{cases}$$

(*)

نقطة انحناء $\rightarrow f'(a) \neq f'_+(a)$

$$m=0 \rightarrow f'_-(a) = f'_+(a) \text{ و } 0 \in$$

$$m \geq 1 \rightarrow \begin{cases} f'_-(a) = 0 \\ f'_+(a) = 0 \end{cases} \rightarrow f'_-(a) = f'_+(a) \text{ و } 0 \in$$

$$m=1 \rightarrow \begin{cases} f'_-(a) = 0 \\ f'_+(a) = 1 \end{cases} \rightarrow f'_-(a) \neq f'_+(a)$$

بمقدار صحيح m

$$g(a), f'(g(a)), (f \circ g)'(a) \Rightarrow g(-\sqrt[3]{2}), f'(g(-\sqrt[3]{2})), (f \circ g)'(-\sqrt[3]{2}) \quad (*)$$

$$f(x) = \frac{1}{\sqrt[3]{x-1}}, g(x) = \frac{1}{x^2-1} \Rightarrow f \circ g = \frac{1}{\sqrt[3]{\frac{1}{x^2-1}-1}} = \frac{1}{\frac{1}{x^2-1}-1} = x$$

بالمركبة از صفر است پس قدر مطلقها را سقش می کنیم

$$\frac{1}{x^2-1} - 1 = \frac{1}{x^2-1} - \frac{x^2-1}{x^2-1} = \frac{1-(x^2-1)}{x^2-1} = \frac{2-x^2}{x^2-1}$$

(*)

$$f \circ g(x) = x \Rightarrow (f \circ g)'(x) = 1 \Rightarrow (f \circ g)'(-\sqrt[3]{2}) = 1 \quad \checkmark$$

$$(1) h(x) = -ax - a \Rightarrow h'(x) = -a \text{ و } h'(a) = -a \quad (*)$$

$$(2) h(x) \text{ و } f(x) \text{ در نقطه } a \text{ مماس هستند} \Rightarrow f'(a) = h'(a) \text{ و } f(a) = h(a) \quad (*)$$

$$\Rightarrow f'(a) = f'(a) = 1 \Rightarrow h'(a) = h'(a) = 1 \Rightarrow -a + 3a + a = 1 \Rightarrow a = \frac{1}{3}$$

$$\frac{f(a)}{f'(a)} = \frac{h(a)}{h'(a)} = \frac{-3a-a}{-a} = \frac{3a+a}{a} = \frac{4a}{a} = 4 = \frac{1}{\frac{1}{3}} = 3 \quad \checkmark$$

