نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

۱

$$f(x)=\begin{cases} a+\sqrt{x^2} & ; x<1 \\ b\sqrt[3]{x^2} & ; x\geq 1 \end{cases} \qquad a+1=b(1) \Rightarrow a+1=b \Rightarrow 1=b-a$$

۲

$$g'(x)=\frac{\left(-1+\frac{2}{2\sqrt{2x}}\right)\times\left(\sqrt[3]{x^2}\right)-\left(\frac{2x}{2\sqrt{x}}\times\left(\sqrt{(x-2)^2}+\sqrt{2x}\right)\right)}{\left(\sqrt[3]{x^2}\right)^2}=\frac{-8-2\sqrt{3}}{3}$$

$$f'(x)=\frac{\left(-2x\sqrt[3]{x^2}\right)-\left(\frac{2x}{2\sqrt{x}}\times(2x-6)\right)}{\left(\sqrt[3]{x^2}\right)^2}=-2-\frac{6}{3}=-\frac{10}{3}$$

$$f'(1)-2g'(3)=-\frac{10}{3}-2\left(\frac{-8-2\sqrt{3}}{3}\right)=\frac{14+4\sqrt{3}}{3}$$

۳

$$f(x)=\begin{cases} bx+c & ; x\leq a \\ \frac{1}{x} & ; x\geq a \end{cases} \qquad ba+c=\frac{1}{a} \Rightarrow -\frac{1}{a^2}\times a+c=\frac{1}{a}$$

$$-\frac{1}{a}+c=\frac{1}{a} \Rightarrow c=\frac{2}{a}$$

$$f'(x)\xrightarrow{x<a} b$$

$$f'(x)\xrightarrow{x>a} x^{-1} \Rightarrow -x^{-2}=-\frac{1}{x^2} \qquad b=-\frac{1}{a^2}$$

$$ac=a\times\frac{2}{a}=2$$

۴

$$f(x)=\begin{cases} b & ; x<a \\ b+(x-a)^m & x\geq a \end{cases}$$

۵

$$g'\left(-\sqrt[3]{2}\right)f'\left(g\left(-\sqrt[3]{2}\right)\right)=\frac{-1}{\sqrt[3]{\frac{1}{16}}}\times\frac{-3\sqrt[3]{2^2}}{8}=\frac{-3\sqrt[3]{2}}{4\sqrt[3]{\frac{1}{2^4}}}=-\frac{3}{4}\times\sqrt[3]{32}$$

$$g(-\sqrt{2})=\frac{1}{-2-1-2}=\frac{-1}{4} \Rightarrow g'(-\sqrt[3]{2})=\frac{1}{2\sqrt[3]{2^2}}=\frac{0-4x}{4x^2}=\frac{-4x}{4x^2}=-\frac{2}{8}\times-\sqrt[3]{2}$$

$$f'\left(-\frac{1}{2}\right) \Rightarrow 2=\frac{1}{\sqrt[3]{2x}} \Rightarrow f'\left(-\frac{1}{2}\right)=\frac{0-\frac{2}{3\sqrt[3]{4x^2}}}{\left(\sqrt[3]{2x}\right)^2}=\frac{-2}{3\sqrt[3]{\frac{1}{2^2}}\times\sqrt[3]{\frac{1}{2^2}}}=\frac{-2}{3\sqrt[3]{\frac{1}{16}}}$$

فرید غازی

۶

$y=-ax-5 \Rightarrow -a(3)-5=-3a-5=f(3) \qquad f'(3)=-a$

$f'(3)-f(3)=2 \Rightarrow -a-(-3a-5)=2 \Rightarrow -a+3a+5=2$

$2a+5=2 \Rightarrow 2a=-3$, $a=-1/5$

$f'(3)=-a=-(-1/5)=1/5 \qquad \frac{f(3)}{f'(3)}=\frac{-0/5}{1/5}=\frac{1}{3}$

$f(3)=-3(-1/5)-5=4/5-5=-0/5$

۷

$fog'(\frac{3}{\sqrt8})=g'(\frac{3}{\sqrt8})\cdot f'(g(\frac{3}{\sqrt8}))$

$g(\frac{3}{\sqrt8})=\frac{1}{\sqrt[3]{x^2-1}}=\frac{1}{\sqrt[3]{\frac98-1}}=\frac{1}{\sqrt[3]{\frac18}}=\frac{1}{\frac12}=2$

$f'(x)=6x^5+4x^3+x=\boxed{226}$

۸

$g(x)=f(x+1)+f(3x+10)\Rightarrow g(-2)=f(-1)+f(4)$

$g'(-2)=f'(-1)+f'(4)=2\times\frac32=3$

چون ۱- و ۴ در یک تناوب هستند مشتق آنها برابر است

۹

$\lim_{h\to0}\frac{f^2(5-h)-3f(5-h)+2}{h(5-h)}=\lim_{h\to0}\frac{f^2(5-h)-3f(5-h)+2}{5h-h^2}$

$\overset{HoP}{\Longrightarrow}\lim_{h\to0}\frac{-2f'(5-h)f(5-h)+3f'(5-h)}{-2h+5}=\frac{-2f'(5)f(5)+3f'(5)}{5}=\frac{-2\times\frac94\times2+3\times\frac94}{5}=\frac{-9}{20}$

$f(x)=(x-4)\sqrt[3]{x+3}\Rightarrow f(5)=1\sqrt[3]{5+3}=2 \qquad f'(5)=\sqrt[3]{x+3}+\frac{(x-4)}{3\sqrt[3]{(x+3)^2}}=\sqrt[3]8+\frac{1}{3\sqrt[3]{64}}=2+\frac14=\frac94$

$f(5)=2$

۱۰

$2y-3x=1\Rightarrow 2y=1+3x\Rightarrow y=1/5x+0/5$

$2x-4=1/5x+0/5\Rightarrow 0/5x=4/5$

$\boxed{x=9}$