

$$f_+(1) = f_-(1) \rightarrow b = a + 1 \rightarrow a = \frac{1}{4}$$

$$f'_+(1) = f'_-(1) \rightarrow b \times \frac{2x}{x\sqrt{x^2}} = \frac{2x}{x\sqrt{x^2}} \rightarrow b \times \frac{2}{x} = 1 \rightarrow b = \frac{3}{4}$$

$$a + b = \frac{1}{4} + \frac{3}{4} = 1$$

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$$f(x) = \frac{x-2x}{\sqrt{x^2}} \quad g(x) = \frac{|x-1| + \sqrt{x}}{\sqrt{x^2}} = \frac{x-x+1+\sqrt{x}}{\sqrt{x^2}}$$

$$f'(1) = \frac{(-1) \times (\sqrt{x^2}) - (x-2x) \left(\frac{2x}{x\sqrt{x^2}} \right)}{(\sqrt{x^2})^2} = \frac{-1 - \frac{2}{x}}{x} = \frac{-1 - \frac{2}{1}}{1} = -3$$

$$g'(1) = \frac{(-1 + \frac{1}{\sqrt{x}}) \times \sqrt{x^2} - (x-x+1+\sqrt{x}) \times \frac{2x}{x\sqrt{x^2}}}{(\sqrt{x^2})^2} = \frac{-1 + \frac{1}{x} - \frac{2}{x} - \frac{1}{x}}{x} = \frac{-1 - \frac{1}{x}}{x} = -\frac{1}{x} - \frac{1}{x^2}$$

$$f'(1) - 2g'(1) = -3 - \left(-\frac{1}{1} - \frac{1}{1^2} \right) = \frac{4\sqrt{3}-9}{3}$$

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$$\left. \begin{aligned} f_+(a) &= f_-(a) \rightarrow \frac{1}{a} = ba + c \rightarrow 1 = ba^2 + ca \\ f'_+(a) &= f'_-(a) \rightarrow \frac{1}{a^2} = b \rightarrow \frac{1}{a^2} = b \rightarrow 1 = ba^2 \end{aligned} \right\} \rightarrow \boxed{ac=0}$$

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$$f'_-(a) = 0$$

$$f'_+(a) = m(x-a)^{m-1} = 0$$

$$f'_-(a) = f'_+(a)$$

میدان مقدار m

در هر مقداری از m، تابع در نقطه a مشتق پذیر است.

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$$f(x) = \frac{1}{\sqrt{x}} \quad g(x) = \frac{1}{x^2}$$

$$g'(x) \times f'(g(x)) = (f \circ g(x))' = \left(\frac{1}{\sqrt{x}} \right)' = -\frac{1}{2} \times x^{-\frac{1}{2}} = -\frac{1}{2} \times \frac{1}{(\sqrt{x})^2} = \boxed{-\frac{1}{4\sqrt{x}}}$$

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$$f(x) = -3a - a = -\frac{19}{4}$$

$$f'(x) = -a = -\frac{3}{4}$$

$$f'(x) - f(x) = 3a + a = 7 \rightarrow a = \frac{3}{4}$$

$$\frac{f(x)}{f'(x)} = \frac{-\frac{19}{4}}{-\frac{3}{4}} = \boxed{\frac{19}{3}}$$

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$$(f \circ g(x))' = g'(x) \times f'(g(x)) = \frac{1}{x} (x^2 - 1)^{-\frac{3}{4}} \times 2g(x) [g'(x) + \frac{1}{x}]^2 \times 2x$$

$$f(x) = x^2 \times [x^2 + \frac{1}{x}]^2 + 1 \rightarrow f'(x) = 2x [x^2 + \frac{1}{x}]^2$$

$$g(\frac{x}{\sqrt{x}}) = x$$

$$(f \circ g(\frac{x}{\sqrt{x}}))' = \frac{1}{x} (\frac{1}{x})^{-\frac{3}{4}} \times 2x \times 2 \times [x + \frac{1}{x}]^2 \times \frac{x}{\sqrt{x}} = \frac{x}{x} \times 2 \times 2 \times 1 \times \frac{x}{\sqrt{x}} = 4\sqrt{x}$$

$$\frac{4\sqrt{x}}{4\sqrt{x}} = \boxed{1}$$

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$$g'(x) = f'(x+1) + 3f'(3x+10) \rightarrow g'(-2) = f'(-1) + 3f'(4) = 4f'(-1) = \boxed{4}$$

$$f'(-1) = f'(4)$$

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$$\lim_{h \rightarrow 0} \frac{(f(a-h)-1)(f(a-h)-2)}{h(a-h)} = \frac{(f(a-h)-1)(f(a-h)-f(a))}{(a-h) \times h}$$

$$f(a) = 2$$

$$\frac{f(a-h)-1}{(a-h)} \times (-f'(a)) = \frac{1}{a} \times (-\frac{2a}{14}) = -\frac{a}{14}$$

$$f'(x) = 1 \times \sqrt[3]{x+3} + (x-4) \times \frac{1}{\sqrt[3]{(x+3)^2}}$$

$$f'(a) = 2 + \frac{1}{14} = \frac{29}{14}$$

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$$y = x^2 - 4x + a$$

$$4y = 3x + 1 \rightarrow y = \frac{1}{4}x + \frac{1}{4} \rightarrow \text{slope} = -2$$

$$y' = 2x - 4 = -2 \rightarrow x = 1 \rightarrow y = 2 \rightarrow \boxed{A|1}$$

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