

چون پیوسته است یعنی مقدار دو طرف با هم برابر است
 $f(x) = \begin{cases} a + \sqrt{x} & x < 1 \\ b\sqrt{x} & x \geq 1 \end{cases}$ $a + 1 = b$
 $a - b = -1$

تابع در $x=0$ تغییر می کند و این همان نقطه مشتق ناپذیری است
 $f(x) = \begin{cases} a + 1/x & x < 1 \\ b\sqrt{x} & x \geq 1 \end{cases}$ $\rightarrow b = \frac{1}{4} \rightarrow a = \frac{1}{4}$ (۲)

$f(x) = \begin{cases} a - x & x \leq 0 \\ a + x & 0 < x \leq 1 \\ b\sqrt{x} & x > 1 \end{cases} \rightarrow f'(x) = \begin{cases} -1 & x \leq 0 \\ 1 & 0 < x \leq 1 \\ \frac{1}{2\sqrt{x}} & x > 1 \end{cases}$ $f'(1) = 1 = \frac{1}{2\sqrt{1}} \rightarrow b = \frac{1}{2}$ (۲)

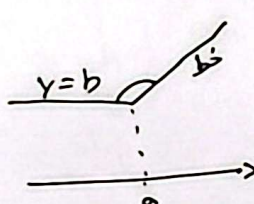
$f(x) = \frac{12x-4}{\sqrt{x}} = \frac{4-2x}{\sqrt{x}} \rightarrow f'(x) = \frac{-2x(\sqrt{x}) - (4-2x)(\frac{1}{2\sqrt{x}})}{(\sqrt{x})^2}$ $x=1 \rightarrow f'(1) = -2 - \frac{4}{2} = -4$

$f'(1) - 2f'(1) = -\frac{10}{3} + \frac{10+12}{3} = \frac{2}{3} = \frac{1}{\sqrt{3}}$ (۲)

$g(x) = \frac{\sqrt{(x-2)^2} + \sqrt{x}}{\sqrt{x}} = \frac{(x-2) + \sqrt{x}}{\sqrt{x}} \rightarrow g'(x) = \frac{(-1 + \frac{1}{2\sqrt{x}})(\sqrt{x}) - (x-2+\sqrt{x})(\frac{1}{2\sqrt{x}})}{(\sqrt{x})^2}$ $\rightarrow g'(1) = -1 + \frac{1}{2\sqrt{1}} - \frac{1}{2\sqrt{1}} = -1$

چون همواره مشتق ناپذیر است
 $f(x) = \begin{cases} bx + c & x < a \\ \frac{1}{x} & x \geq a \end{cases}$ $x=a \rightarrow ab + c = \frac{1}{a}$ (۱)
 مشتق $\rightarrow b = -\frac{1}{a^2} \Rightarrow b = -\frac{1}{a^2}$ (۲)

(۱) و (۲) $\rightarrow -\frac{1}{a} + c = \frac{1}{a} \rightarrow \frac{c}{a} = \frac{2}{a} \rightarrow ac = 2$ (۲)



اگر $m > 0$ مشتق را می توانیم
 $f(x) = \begin{cases} b & x < a \\ b + m(x-a) & x \geq a \end{cases}$ $m=1$ (۲)

اگر $m=0$ تابع در $x=a$ مشتق ناپذیر است
 $f'(x) = \begin{cases} 0 & x < a \\ m & x \geq a \end{cases}$ $m=1$ (۲)

$g'(c-\sqrt{x}) f'(g(c-\sqrt{x})) = (f \circ g(c-\sqrt{x}))' \rightarrow x = -\sqrt{x} \rightarrow f(x) = \frac{1}{\sqrt{x}}$

$f \circ g(x) = f(\frac{1}{\sqrt{x}}) = \frac{1}{\sqrt{\frac{1}{\sqrt{x}}}} = \sqrt{x} \rightarrow (f \circ g)' = (\sqrt{x})' = \frac{1}{2\sqrt{x}}$ $g(x) = \frac{1}{\sqrt{x}}$ (۲)

$g'(1) = -1 + \frac{1}{2\sqrt{1}} - \frac{1}{2\sqrt{1}} = -1$

$$f(x) = -x^2 - 2, f'(x) = -2x \quad \xrightarrow{f'(x) - f(x) = 2} -2x + x^2 + 2 = 2 \rightarrow x = \frac{2}{x} \rightarrow x = \sqrt{2}$$

$$f(x) = -x^2 - \frac{2}{x} - 2 = -\frac{1}{x} \rightarrow \frac{f(x)}{f'(x)} = \frac{-\frac{1}{x}}{\frac{2}{x}} = -\frac{1}{2} \quad \checkmark$$

$$f'(x) = \frac{2}{x}$$

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$$g\left(\frac{x}{\sqrt{2}}\right) = \frac{1}{\sqrt{\frac{x}{\sqrt{2}} - 1}} = 2 \rightarrow f \circ g\left(\frac{x}{\sqrt{2}}\right) = g\left(\frac{x}{\sqrt{2}}\right) \times f\left(g\left(\frac{x}{\sqrt{2}}\right)\right) \quad (f \circ g(x))' = g'(x) \times f'(g(x))$$

$$g(x) = (x^2 - 1)^{-\frac{1}{2}} \rightarrow g'(x) = -\frac{1}{2}(x^2 - 1)^{-\frac{3}{2}} \times 2x = \frac{-x}{\sqrt{(x^2 - 1)^3}} \rightarrow g\left(\frac{x}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

$$f(x) = (x^2)^2 + 1 \rightarrow f'(x) = 4x^3 \rightarrow f'(x) = 4x^3$$

$$\Rightarrow f \circ g\left(\frac{x}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}} \times 4x^3 = -\frac{4x^3}{\sqrt{2}} \rightarrow \frac{-4x^3}{\sqrt{2}} = -2\sqrt{2}x^3 \quad \checkmark$$

در این مرحله

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$$g(x) = f(x+1) + f(x+2)$$

$$g'(x) = f'(x+1) + 2f'(x+2)$$

$$g'(1) = f'(2) + 2f'(3)$$

$$g'(1) = \frac{2}{1} + \frac{2}{2} = \frac{3}{1} = 3 \quad \checkmark$$

و در مرحله بعد

$$f(1) = f(2)$$

$$f(1) = f'(2) = \frac{2}{1}$$

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$$\lim_{h \rightarrow 0} \frac{(f(a-h))^2 - 2f(a-h) + 1}{a-h} \stackrel{\frac{0}{0}}{=} \lim_{h \rightarrow 0} \frac{2(-f(a-h))f'(a-h) + 2f'(a-h)}{a-h}$$

$$\stackrel{h=0}{=} \frac{-2f'(a)f(a) + 2f'(a)}{a} = -\frac{f'(a)}{a} = -\frac{\frac{2a}{12}}{\frac{a}{1}} = -\frac{2}{12} \quad \checkmark$$

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$$f(a) = 2 \quad f'(x) = 1 \times \sqrt{x+2} + (x-2) \times \frac{1}{2\sqrt{x+2}} \rightarrow f'(a) = 2 + \frac{1}{12} = \frac{25}{12}$$

$$y = 2x + 1 \rightarrow y = \frac{1}{2}x + \frac{1}{2} \rightarrow m = \frac{1}{2} \rightarrow m' = -2$$

در این مرحله $y = x^2 - 2x + 2$ باید نقطه ای را پیدا کنیم که در آن $y = 2$ باشد

$$y' = 2x - 2 = -2 \rightarrow 2x = 2 \rightarrow x = 1$$

$$y = x^2 - 2x + 2 \quad x=1 \rightarrow y = 1 - 2 + 2 = 1 \rightarrow \text{نقطه } (1, 1) \quad \checkmark$$

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