۱- $g'(x) = \left(2(1+\tan^2 x)\tan x - \sqrt{2}\sin x\right) f'\left(\tan^2 x + \sqrt{2}\cos x\right)$

$g'\left(\frac{\pi}{4}\right) = (4-1)f'(2) \Rightarrow \frac{\sqrt{3}}{3} = f'(2)$

۲- $f(x) = \frac{(2+\cos x)(4 - 2\cos x + \cos^2 x)}{(2+\cos x)(2-\cos x)} = \frac{4 - 2\cos x + \cos^2 x}{2 - \cos x}$

$f' - 2g'\left(\frac{7\pi}{6}\right) = (f - 2g)'\left(\frac{7\pi}{6}\right)$

$f - 2g(x) = \frac{-2\cos x + \cos^2 x}{2 - \cos x} = -\cos x$

$\Rightarrow (f - 2g)'(x) = \sin x \Rightarrow (f-2g)'\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

۳-

$\left| f'_+\left(\frac{3}{4}\right) - f'_-\left(\frac{3}{4}\right) \right| = 2\sqrt{6}$

$\Rightarrow 4\sqrt{\frac{3}{4}a} - (-4)\sqrt{\frac{3}{4}a} = 8\sqrt{\frac{3}{4}a} \Rightarrow a = \frac{1}{2}$

۴- $f(4) = 2 - 4a$

$f'(4) = -a$

$2 - 4a + (-a) = -1 \Rightarrow a = \frac{3}{5}$

$\Rightarrow f'(4) = -\frac{3}{5}$

۵- $y = \frac{x+5}{7}$

$y' = \frac{1}{7}$

$y' = \frac{a+3}{(3x+1)^2}$

$\frac{1}{7} = \frac{a+3}{(3x+1)^2} \xRightarrow{x=2} a = 4$

۶-

$f(۲)=0 \Rightarrow ۸+۲a-b=0$

$f'(۲)=0 \Rightarrow ۳x^۲+a,\ x=۲ \Rightarrow ۱۲+a=0 \Rightarrow a=-۱۲,\ b=-۱۶$

$b-a=-۴$

۷-

$f'(x)=۲n\times x\times \cos x^۲ \times \sin^{n-1} x^۲$

$$\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m} \overset{\text{هم‌ارزی}}{=} \lim_{x\to 0}\frac{۲n\times x^{۴n-1}}{\frac{x^{۲m}}{۲^m}}=۲^{\frac{۱۱}{۲}}$$

$n\times ۲^{m+1}=۲^{\frac{۱۱}{۲}} \qquad ۴n-1=۲m \Rightarrow n=۲,\ m=\frac{۷}{۲} \Rightarrow ۲m+n=۹$

۸-

$$f'(x)=\cancel{\frac{1}{(f^{-1})'(x)}}\ \frac{1}{(f^{-1})'(f(x))}$$

$$f'(x)=\frac{\left(\frac{1}{۲\sqrt{x}}\right)(\sqrt{x}-1)-(\sqrt{x}+1)\left(\frac{1}{۲\sqrt{x}}\right)}{(\sqrt{x}-1)^۲}$$

$\cancel{f'(۲)} \Rightarrow f(x)=۲ \Rightarrow x=۹$

$\Rightarrow f'(۹)=-\frac{1}{۱۲} \Rightarrow (f^{-1})'(۲)=-۱۲$

۹-

$f(-1)=-۸a+۸ \qquad \frac{1-(-۸a+۸)}{1}=-۱۱ \Rightarrow a=\frac{۹}{۴}$

$f(0)=1$

۱۰-

$$\frac{\sin\frac{\pi}{۲}\cos\pi-\sin\frac{\pi}{۴}\cos\frac{۳\pi}{۲}}{\sin^۴\frac{\pi}{۲}-\cos^۴\frac{\pi}{۲}-\left(\sin^۴\frac{\pi}{۴}-\cos^۴\frac{\pi}{۴}\right)}=\frac{-1}{1}=-1$$