

$$g'(x) = (2 \tan x (1 + \tan^2 x) - \sqrt{x} \sin x) f'(\tan x + \sqrt{x} \cos x)$$

$$x = \frac{\pi}{4} \rightarrow g'\left(\frac{\pi}{4}\right) = 3f'(1) = \sqrt{3} \rightarrow \boxed{f'(1) = \frac{\sqrt{3}}{3}}$$

$$f(x) = \frac{1 + \cos^2 x}{1 - \cos^2 x} = \frac{(1 + \cos x)(1 - \cos x)}{(1 - \cos x)(1 + \cos x)} = \frac{\cos^2 x - \cos x + 1}{1 - \cos x} \rightarrow f'(x) = \frac{\sin x \cos x (\cos x - 1)}{(1 - \cos x)^2}$$

$$g(x) = \frac{x}{1 - \cos x} = x(1 - \cos x)^{-1} \rightarrow g'(x) = -\frac{x \sin x}{(1 - \cos x)^2}$$

$$\rightarrow f'\left(\frac{\sqrt{19}}{4}\right) - 2g'\left(\frac{\sqrt{19}}{4}\right) = -\frac{5 + 11\sqrt{19}}{2(1 + \sqrt{19})^2} - \frac{1}{(1 + \sqrt{19})^2} = \boxed{\frac{-19 - 11\sqrt{19}}{2(1 + \sqrt{19})^2}}$$

$$f(x) = |x^2 - 2| \sqrt{a} x \rightarrow f(x) = \begin{cases} (x^2 - 2) \sqrt{a} x, & x < \frac{\pi}{4} \\ (2 - x^2) \sqrt{a} x, & x > \frac{\pi}{4} \end{cases}$$

$$f'_-(x) = \frac{(x^2 - 2)a}{2\sqrt{a}x} - \sqrt{a}x \rightarrow f'_-\left(\frac{\pi}{4}\right) = -\sqrt{\frac{x}{a}} = -2\sqrt{a} \quad f'_+\left(\frac{\pi}{4}\right) - f'_-\left(\frac{\pi}{4}\right) = 4\sqrt{a} = 2\sqrt{4}$$

$$f'_+(x) = \frac{(2 - x^2)a}{2\sqrt{a}x} + \sqrt{a}x \rightarrow f'_+\left(\frac{\pi}{4}\right) = \sqrt{\frac{x}{a}} = 2\sqrt{a} \Rightarrow \sqrt{a} = \frac{\sqrt{4}}{2} = \frac{\sqrt{2}}{2} \Rightarrow 2a = \frac{x}{2} \Rightarrow \boxed{a = \frac{1}{2}}$$

$$y = -ax + 2 \rightarrow f(f) = -fa + 2, \quad f'(f) = -a$$

$$f(f) + f'(f) = -fa + 2 - a = -1 \rightarrow a = \frac{x}{6} \Rightarrow f'(f) = -a = \boxed{\frac{-x}{6}}$$

$$\begin{cases} y = \frac{x+5}{y} \\ y = \frac{ax-1}{2x+1} \end{cases} \rightarrow \frac{x+5}{y} = \frac{ax-1}{2x+1} \rightarrow 2x^2 + (14 - 1a)x + 12 = 0$$

چون معادله ریشه مضاعف دارد می توان آن را به صورت مربع کامل نوشت و چون محل تلاقی نایه اول است معادله به صورت $3(x-2)^2$ است.

$$3(x-2)^2 = 3x^2 - 12x + 12 = 0 \rightarrow 14 - 1a = -12 \Rightarrow 1a = 26 \Rightarrow \boxed{a = 26}$$

$2=1$ هر رله دوم اکثره تابع است

$$f'(x) = 2x + a \Rightarrow f'(1) = 1 + a = 0 \Rightarrow \underline{a = -1}$$

$$f(1) = 1 + 2a - b = 1 - 2 - b = 0 \Rightarrow \underline{b = -1}$$

$$\Rightarrow b - a = -1 + 1 = \underline{0}$$

$$f(x) = \sin^n(x) \Rightarrow f'(x) = n \sin^{n-1}(x) \times \cos(x) \times 1 \quad \begin{matrix} \sin(x) \sim x \\ \cos(x) \sim 1 \end{matrix}$$

$$f(x)f'(x) = (n \sin^{n-1}(x) \cos(x)) \sin^n(x) \stackrel{x \rightarrow 0}{\sim} n x^{2n-1} = n x^{2n-1}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)f'(x)}{(1-\cos x)^m} = \frac{n x^{2n-1}}{x^{2m}} = n x^{2n-1-2m} = n x^k \Rightarrow 2n-1-2m=0 \Rightarrow 2m=2n-1$$

$$\Rightarrow n x^k = n x^{\frac{2n-1}{2}} = x^{\frac{2n-1}{2}} \Rightarrow k = \frac{2n-1}{2} \Rightarrow 2m = 2n-1 \Rightarrow m = \frac{2n-1}{2}$$

$$y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x}y - y = \sqrt{x}+1 \Rightarrow \sqrt{x}y - \sqrt{x} = y+1 \Rightarrow \sqrt{x}(y-1) = y+1$$

$$\sqrt{x} = \frac{y+1}{y-1} \Rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \Rightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$$

$$\Rightarrow (f^{-1}(x))' = 2 \left(\frac{x+1}{x-1}\right) \left(\frac{1(x-1) - 1(x+1)}{(x-1)^2}\right) \stackrel{x=1}{\rightarrow} m = 2(2)(-2) = \underline{-8}$$

$$[1-1] = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{1 - (1-a)}{2} = \frac{1-a-1}{2} = \frac{-a}{2} = -1 \Rightarrow \underline{a = 2}$$

$$\Rightarrow f(x) = (x+1)^2 \left(-\frac{1}{2}x+1\right) \Rightarrow f'(x) = 2(x+1) \left(-\frac{1}{2}x+1\right) - \frac{1}{2}(x+1)^2$$

$$x = -2a = 1 \Rightarrow f'(1) = (2 \times 1 \times \frac{1}{2}) - \frac{1}{2} = 1 - \frac{1}{2} = \underline{\frac{1}{2}}$$

$$f(x) = \sin x \cos x \Rightarrow \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{\sin \frac{\pi}{4} \cos \frac{\pi}{4} - \sin \frac{\pi}{2} \cos \frac{\pi}{2}}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{-1}{\frac{\pi}{4}} = \underline{-\frac{4}{\pi}}$$

$$g(x) = \sin^2 x - \cos^2 x = (\sin x + \cos x)(\sin x - \cos x) = \sin^2 x - \cos^2 x = -\cos^2 x$$

$$\Rightarrow \frac{g(\frac{\pi}{4}) - g(\frac{\pi}{2})}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{-\cos^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{2}}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{\frac{1}{4}}{\frac{\pi}{4}} = \underline{\frac{1}{\pi}}$$