نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس ...... A ....

۱	$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$, $g'(\frac{\pi}{4}) = \sqrt{3}$ $g(\frac{\pi}{4}) = f(1^2 + \sqrt{2}\cdot\frac{\sqrt{2}}{2})$, $f(2)$ $\Rightarrow g'(\frac{\pi}{4}) = f'(2)\cdot 2\tan x \sec^2 x - \sqrt{2}\sin x$, $2\times(1)(2) - \sqrt{2}\cdot\frac{\sqrt{2}}{2}$ $\rightarrow g'(\frac{\pi}{4}) = f'(2)\times 3$, $\sqrt{3}$ $\nearrow f'(2) = \frac{\sqrt{3}}{3}$
۲	$f(x) = \frac{8+\cos^2 x}{4-\cos^2 x} \Rightarrow f'(x) = \frac{(-2\cos x\cdot\sin x)(4-\cos^2 x) - (8+\cos^2 x)(2\cos x\sin x)}{(4-\cos^2 x)^2}$ $g(x) = \frac{2}{2-\cos x} \Rightarrow g(x) = 2(2-\cos x)^{-1} \Rightarrow g'(x) = 2(-1)(2-\cos x)^{-2}\cdot\sin x$ حاصل نهایی $= \frac{3\sqrt{3}}{2} - \sqrt{3} = \frac{\sqrt{3}}{2}$
۳	$f'_+(\frac{3}{4})$, $f'_-(\frac{3}{4}) \rightarrow$ این ۲ مقدار قرینه اند پس $\leftarrow 2, f'_+(\frac{3}{4}) = 2\sqrt{9}$ $(4x-3)\sqrt{ax}$ $\quad (-4x+3)\sqrt{ax}$ $\Rightarrow ((4x-3)\sqrt{ax})' \rightarrow 4\sqrt{ax} + (4x-3)\cdot\frac{a}{2\sqrt{ax}}$ $\Rightarrow \frac{8ax+4ax-3a}{2\sqrt{ax}} = \sqrt{9} \Rightarrow 3a(\cancel{4x}-1) = 2\sqrt{9a\cdot\frac{3}{4}}$, $x=\frac{3}{4}$ $\Rightarrow 36a^2 = 18a \Rightarrow a = \frac{1}{2}$
۴	$y = -ax+2 \rightarrow y' = -a$, $f(4) = -4a+2$ $\left.\begin{matrix} f'(4) + f(4) = -1 \\ -4a+2 - a = -1 \end{matrix}\right\} \Rightarrow a = \frac{3}{5}$, $f'(4) = \frac{-3}{5}$
۵	$\left.\begin{matrix} 7y = x+5 \Rightarrow y = \frac{x+5}{7} \\ y = \frac{ax-1}{3x+1} \end{matrix}\right\} \Rightarrow \frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow 3x^2+16x+5 = 7ax-7$ $\hookrightarrow 3x^2 + (16-7a)x + 12 = 0$ باید ریشه مضاعف داشته باشیم $\Rightarrow 3(x-2)^2 = 0$ $\quad 16-7a = -12$ $\boxed{a = 4}$

$f(x) = x^3 + ax - b \Rightarrow f'(x) = 3x^2 + a \underset{\text{[illegible]}}{\Rightarrow} 0 = 3x^4 + a \Rightarrow a = -12$ $x^3 - 12x - b \xRightarrow{x=2} 8 - 24 - b = 0 \Rightarrow \boxed{b = -16}$ $b - a = -16 - (-12) = -4$	۶
$f(x) = \sin^n(x^2) \sim x^{2n} \quad f'(x) \sim 2n u^{2n-1} \Rightarrow f(x) \cdot f'(x) = u^{2n} \cdot 2n x^{2n-1} = 2n u^{4n-1}$ $(1-\cos x)^m \sim \left(\frac{x^2}{2}\right)^m = \frac{x^{2m}}{2^m} , \frac{2n x^{4n-1}}{\frac{x^{2m}}{2^m}} = 2n \cdot 2^m x^{4n-1-2m}$ توان صفر می‌شود $\Rightarrow 2m = 4n-1 , 2n \cdot 2^m = 32\sqrt{2} = 2^5 \cdot 2^{\frac{1}{2}} \Rightarrow n = 2 , m = \frac{7}{2}$ $2m + n = 9$	۷
$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1}(x) \Rightarrow y\sqrt{x} - y = \sqrt{x} + 1 \Rightarrow y\sqrt{x} - \sqrt{x} = y + 1$ $\Rightarrow \sqrt{x}(y-1) = y+1$ $\sqrt{x} = \frac{y+1}{y-1}$ $2 \times (3) \times \left(\frac{-2}{1}\right) \xLeftarrow{x=2} 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) \xleftrightarrow{\text{مشتق زنجیره‌ای}} f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$ $\Rightarrow$ حاصل $= -12$	۸
$\frac{f(b) - f(a)}{b - a} = \frac{1 + 8(a-1)}{0-(-1)} = -11 \Rightarrow 8(a-1) = -12$ $a - 1 = \frac{-3}{2}$ $f(0) = 1 , f(-1) = 8(-a+1)$ $a = \frac{-1}{2}$	۹
$y_2 = \sin^4 x - \cos^4 x = (\underbrace{\sin^2 x + \cos^2 x}_{1})(\sin^2 x - \cos^2)$ $y_1 \Rightarrow \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{-1}{\frac{\pi}{4}}$ $y_2 \Rightarrow \frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{\frac{\pi}{4}} = \frac{1}{\frac{\pi}{4}}$ $\Rightarrow$ حاصل $= \frac{\frac{-1}{\frac{\pi}{4}}}{\frac{1}{\frac{\pi}{4}}} = -1$	۱۰