SUBJECT:

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آراد زادنیا — تکلیف شماره ۲۳ — دوازدهم پسر A

۱)

$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \Rightarrow g'(x) = \left(2\tan x \times \frac{1}{\cos^2 x} - \sqrt{2}\sin x\right) f'(\tan^2 x + \sqrt{2}\cos x)$

$\Rightarrow g'(\frac{\pi}{4}) = (2\times 2 - 1) f'(2) = \sqrt{3} \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$

۲)

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{(2+\cos x)(\cos^2 x - 2\cos x + 4)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$

$g(x) = \frac{2}{2-\cos x} \Rightarrow f(x) - 2g(x) = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x} - \frac{4}{2-\cos x} = \frac{\cos^2 x - 2\cos x}{2-\cos x}$

$= \frac{\cos x(\cos x - 2)}{2-\cos x} = -\cos x \Rightarrow (-\cos x)' = \sin x \xrightarrow{x=\frac{7\pi}{6}} \sin\frac{7\pi}{6} = -\frac{1}{2}$

۳)

$f(x) = |4x-3|\sqrt{ax}$

$x > \frac{3}{4}$: $(4x-3)\sqrt{ax} \Rightarrow f'(x) = 4\sqrt{ax} \xrightarrow{x=\frac{3}{4}} 2\sqrt{3a}$

$x < \frac{3}{4}$: $(3-4x)\sqrt{ax} \Rightarrow f'(x) = -4\sqrt{ax} \xrightarrow{x=\frac{3}{4}} -2\sqrt{3a}$

اختلاف شیب ها $= 2\sqrt{3a} - (-2\sqrt{3a}) = 4\sqrt{3a} = 2\sqrt{6} \Rightarrow a = \frac{1}{2}$

۴)

$y = -ax + 2 \Rightarrow y' = -a \xrightarrow{\text{مماس در نقطه } x=4} f'(4) = -a \;,\; f(4) = -4a + 2$

$\Rightarrow f(4) + f'(4) = -5a + 2 = -1 \Rightarrow a = \frac{3}{5} \Rightarrow f'(4) = -\frac{3}{5}$

۵)

$7y - x = 5 \Rightarrow y = \frac{x+5}{7} \Rightarrow \frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow 3x^2 + 16x + 5 = 7ax - 7$

$\Rightarrow 3x^2 + (16-7a)x + 12 = 0 \Rightarrow x^2 + \frac{16-7a}{3}x \xrightarrow{x>0} \frac{16-7a}{3} = -4$

$\Rightarrow 16 - 7a = -12 \Rightarrow 7a = 28 \Rightarrow a = 4$

Celine

(۶) مماس بر محور x

$f(x)=x^3+ax-b \Rightarrow f'(x)=3x^2+a \ , \ f'(2)=0$

$\Rightarrow 3(2)^2+a=0 \Rightarrow a=-12 \ , \ f(2)=0 \Rightarrow f(2)=3(2)^2-12(2)-b=0$

$\Rightarrow b=-16 \Rightarrow b-a=-16-(-12)=-4$

(۷)

$f(x)=\sin^n(x^2) \Rightarrow f'(x)=2nx\sin^{n-1}(x^2)(\cos x^2)$

$\Rightarrow \lim\limits_{x\to 0}\dfrac{f(x)f'(x)}{(1-\cos x)^m}=\dfrac{(2nx\cos x^2)(\sin^{2n-1}x^2)}{(1-\cos x)^m} \xrightarrow{\text{هم ارزی}}$

$2n+m=7+2=\boxed{9}$

$\dfrac{2nx(x^2)^{2n-1}}{\dfrac{x^{2m}}{2^m}}=\dfrac{(2n\times 2^m)(x^{4n-1})}{x^{2m}}=7 \ , \ \underbrace{2m}_{7}=4n-1 \Rightarrow n=2$

n را ۲ میگذاریم و m را بدست می‌آوریم و از m به n می‌رسیم اگر نشد

$n\times 2^{m+1}=2^{5/5} \Rightarrow n=2 \Rightarrow m+2=5/5 \Rightarrow m=3/5 \Rightarrow 2m=7$

(۸)

$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2 \Rightarrow (f^{-1}(x))'=2\left(\dfrac{-2}{(x-1)^2}\right)\left(\dfrac{x+1}{x-1}\right)$

$\Rightarrow (f^{-1}{}'(2))=2\left(\dfrac{-2}{1}\right)(3)=-12$

(۹)

آهنگ تغییر متوسط تابع در بازه $[-1,0]$ $=\dfrac{f(0)-f(-1)}{0-(-1)}=\dfrac{8a-7}{1}=-11 \Rightarrow a=-\dfrac{1}{2} \Rightarrow$

$f(x)=(x^2+1)^3(-\frac{1}{2}x+1)$

$\Rightarrow f'(x)=6x(x^2+1)^2(-\frac{1}{2}x+1)-\frac{1}{2}(x^2+1)^3 \Rightarrow f'(1)=\underbrace{6\times4\times\frac{1}{2}}_{12}-\underbrace{\frac{1}{2}\times8}_{4}=\underline{8}$

(۱۰)

$f(x)=\sin x\cos 2x=\sin x(1-2\sin^2x)=-2\sin^3x+\sin x$

$g(x)=\sin^4x-\cos^4x=(\sin^2x-\cos^2x)(\cancel{\sin^2x+\cos^2x})=\sin^2x-(1-\sin^2x)$

$=2\sin^2x-1$

نسبت آهنگ تغییر تابع $\longrightarrow \dfrac{\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{4}}}=\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{g(\frac{\pi}{2})-g(\frac{\pi}{4})}=\dfrac{-1-0}{1-0}=\boxed{-1}$