A سری ۳

مشتق بهرامی

1

$$g'(x) = \left(2\tan x(\tan^2 x + 1) - \sqrt{2}\sin x\right) f'(\tan^2 x + \sqrt{2}\cos x)$$

$$x=\frac{\pi}{4}: \; g'\left(\frac{\pi}{4}\right) = \Big(2\underbrace{\tan\frac{\pi}{4}}_{1}\big(\underbrace{\tan^2\frac{\pi}{4}}_{1}+1\big) - \sqrt{2}\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\Big) f'\Big(\underbrace{\tan^2\frac{\pi}{4}}_{1} + \sqrt{2}\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt{2}}{2}}\Big) = (4-1)\, f'(2) = f'(2)$$

$$\Rightarrow f'(2) = \frac{\sqrt{3}}{3}$$

2

$$f'(x) - 2g'(x) = (f-2g)'(x) \quad , \quad f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{(\cos x+2)(\cos^2 x - 2\cos x + 4)}{-(\cos x + 2)(\cos x - 2)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$$

$$(f-2g)(x) = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x} - 2\left(\frac{2}{2-\cos x}\right) = \frac{\cos^2 x - 2\cos x + 4 - 4}{2-\cos x} = \frac{\cos^2 x - 2\cos x}{2-\cos x} = \frac{-\cos x(2-\cos x)}{2-\cos x}$$

$$\Rightarrow (f-2g)(x) = -\cos x \Rightarrow (f-2g)'(x) = \sin x \Rightarrow \boxed{(f-2g)'\left(\frac{7\pi}{6}\right) = \sin\frac{7\pi}{6} = -\frac{1}{2}}$$

3

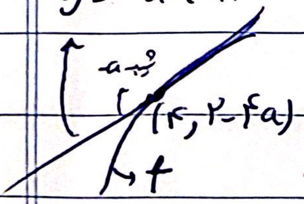
$$f(x) = \begin{cases} (4x-3)\sqrt{ax} \; ; & x \geq \frac{3}{4} \\ (3-4x)\sqrt{ax} \; ; & x < \frac{3}{4} \end{cases} \Rightarrow f'(x) = \begin{cases} 4\sqrt{ax} + \left(\frac{a}{2\sqrt{ax}}\right)(4x-3) \; ; & x > \frac{3}{4} \\ -4\sqrt{ax} + \left(\frac{a}{2\sqrt{ax}}\right)(3-4x) \; ; & \frac{3}{4} > x > 0 \end{cases}$$

$$f'_+\left(\frac{3}{4}\right) = 4\sqrt{a\times\frac{3}{4}} + 0 \;,\; f'_-\left(\frac{3}{4}\right) = -4\sqrt{a\times\frac{3}{4}} + 0 \Rightarrow \left|f'_+\left(\frac{3}{4}\right) - f'_-\left(\frac{3}{4}\right)\right| = \left|4\sqrt{\frac{3a}{4}} - \left(-4\sqrt{\frac{3a}{4}}\right)\right|$$

$$\Rightarrow \left|f'_+\left(\frac{3}{4}\right) - f'_-\left(\frac{3}{4}\right)\right| = \Big|\underbrace{2\sqrt{3a} + 2\sqrt{3a}}_{\text{جمع رادیکال‌های همراه متشابه است}}\Big| = 4\sqrt{3a} = 2\sqrt{6} \Rightarrow 2\sqrt{3a} = \sqrt{6} \Rightarrow \sqrt{a} = \frac{\sqrt{2}}{2} \Rightarrow \boxed{a = \frac{1}{2}}$$

4

$y = -ax + 2 \rightarrow$ شیب $= -a$ , عرض نقطه تماس با طول ۴ $= -4a + 2 = 2 - 4a$




$$\Rightarrow f'(4) = a \;,\; f(4) = 2 - 4a \Rightarrow f(4) + f'(4) = -1 \Rightarrow 2 - 4a - a = -1$$

$$\Rightarrow 5a = 3 \Rightarrow a = \frac{3}{5} \Rightarrow \boxed{f'(4) = -\frac{3}{5}}$$

5

$$\frac{x+a}{7} = \frac{ax-1}{3x+1} \Rightarrow 3x^2 + 16x + a = 7ax - 7 \Rightarrow 3x^2 + (16-7a)x + 12 = 0$$

$$\Rightarrow x^2 + \left(\frac{16-7a}{3}\right)x + 4 = 0$$

معادله برخورد دو منحنی در نقاطی که برخورد دارند:
اگر در آن نقطه مماس نیز بوده باشند
$\Delta = 0$ و عبارت مربع کامل باشد

$\rightarrow (x+2)^2 = 0 \Rightarrow x = -2$ غ ق ق $(x \geq 0)$

$\rightarrow (x-2)^2 = 0$ ق ق $\Rightarrow x^2 - 4x + 4 = 0$

$$\Rightarrow \frac{16-7a}{3} = -4 \Rightarrow 16 - 7a = -12 \Rightarrow 7a = 28 \Rightarrow \boxed{a = 4}$$

(6)

$f(x)=x^3+ax-b \quad , y=0 \xrightarrow[x=2]{\text{مماس}} \begin{cases} f(2)=8+2a-b=0 \\ f'(2)=12+a=0 \end{cases} \Rightarrow \underbrace{f(2)-f(2)}_{0}=4+a+b \Rightarrow \boxed{b-a=-4}$

$\Rightarrow f'(x)=3x^2+a \quad , y'=0$

(7)

~~$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1}(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}=f(x)$~~ ~~در اینجا اگر دقت کنید تابع شکل کلی $y=\frac{ax+b}{cx+d}$ که اگر $a+d=0$ آنگاه~~

~~معکوس خودشان را دارند.~~

~~$f(x)=f^{-1}(x)=g(\sqrt{x}) \Rightarrow f'(x)=\frac{1}{2\sqrt{x}}g'(\sqrt{x})$~~ ~~فرض کنید $g(x)=\frac{x+1}{x-1}$~~

~~$g'(x)=\frac{-1-1}{(x-1)^2}=-\frac{2}{(x-1)^2} \Rightarrow g'(\sqrt{x})=-\frac{2}{(\sqrt{x}-1)^2}$~~ ~~$f'(x)=\frac{1}{2\sqrt{x}}\left(-\frac{2}{(\sqrt{x}-1)^2}\right)=-\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$~~

پاسخ صفحه بعد

~~$f'(2)=-\frac{1}{\sqrt{2}(\sqrt{2}-1)^2}=-\frac{1}{\sqrt{2}(3-2\sqrt{2})} \Rightarrow \boxed{\frac{1}{4-3\sqrt{2}}}$~~

(8)

$f'(x)=2x\,x\sin^{n-1}(x^2)\cos x^2 \Rightarrow f(x)f'(x)=2xx\sin^{n}(x^2)\sin^{n-1}(x^2)\cos(x^2)$

$\text{حاصل حد}=\lim_{x\to 0}\frac{2xx\sin^{2n-1}(x^2)\overbrace{\cos(x^2)}^{1}}{(1-\cos x)^m}=\lim_{x\to 0}\frac{2xx\times(x^2)^{2n-1}}{(1-\cos x)^m}=\lim_{x\to 0}\frac{2x x^{4n-1}}{(1-\cos x)^m}=\lim_{x\to 0}\frac{2xx^{4n-1}}{(1-(1-\frac{1}{2}x^2))^m}$

$=\lim_{x\to 0}\frac{2xx^{4n-1}}{(\frac{1}{2})^m x^{2m}}=32\sqrt{2}$ ؟ $\Rightarrow 4n-1=2m \Rightarrow \boxed{m=2n-\frac{1}{2}}$

چون حاصل حد عدد شده پس درجه صورت و مخرج برابر بوده تا رفع ابهام انجام شود و حاصل عددی مخالف صفر شد!

~~$\Rightarrow 4n=32\sqrt{2} \Rightarrow n=8\sqrt{2} \quad , 4n-1=2m \Rightarrow 32\sqrt{2}-1=2m$~~

~~$m-n=(32\sqrt{2}-1)-(8\sqrt{2})=\boxed{24\sqrt{2}}$~~

$\lim_{x\to 0}\frac{2n\times 2^m\, x^{4n-1}}{x^{2m}}=2n\times 2^m=32\sqrt{2} \Rightarrow n\times 2^m=16\sqrt{2} \Rightarrow n\times 2^{2n-\frac{1}{2}}=2^{\frac{9}{2}} \Rightarrow n=2 \Rightarrow 2m=7 \Rightarrow 2m+n=7+2=9.$

(9)

$f(0)=1 \ , f(-1)=8-8a \Rightarrow \underset{[-1,0]}{\text{آهنگ تغییرات متوسط}}=\frac{f(0)-f(-1)}{0-(-1)}=f(0)-f(-1)=1+8a-8=-11$

$\Rightarrow \boxed{a=-\frac{1}{2}} \Rightarrow f(x)=(x^2+1)^3(-\frac{1}{2}x+1) \Rightarrow f'(x)=3(x^2+1)^2(2x)(-\frac{1}{2}x+1)+(-\frac{1}{2})(x^2+1)^3$

$\Rightarrow x=-2a=1 \rightsquigarrow f'(1)=3\times 2^2\times 2\times\frac{1}{2}-\frac{1}{2}\times 2^3=12-4=\boxed{8}$

$f(x) = \sin x \cos 2x \Rightarrow f(\frac{\pi}{4}) = 0,\ f(\frac{\pi}{2}) = -1 \Rightarrow$ آهنگ تغییرات $f$ $= \dfrac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{-1-0}{\frac{\pi}{4}} = -\dfrac{4}{\pi}$ ۱۵

$g(x) = \sin^4 x - \cos^4 x = \underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = -\cos 2x \Rightarrow g(\frac{\pi}{2}) = 1,\ g(\frac{\pi}{4}) = 0$

آهنگ تغییرات $g$ $= \dfrac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{1-0}{\frac{\pi}{4}} = \dfrac{4}{\pi} \Rightarrow$ $\boxed{\dfrac{\text{آهنگ تغییرات } f}{\text{آهنگ تغییرات } g} = \dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1}$

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$y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y\sqrt{x} - y = \sqrt{x} + 1 \Rightarrow (y-1)\sqrt{x} = y+1 \Rightarrow \sqrt{x} = \dfrac{y+1}{y-1} \Rightarrow x = \left(\dfrac{y+1}{x-1}\right)^2$ ۸

$\Rightarrow f^{-1}(x) = \left(\dfrac{x+1}{x-1}\right)^2 \Rightarrow (f^{-1})'(x) = 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right) \Rightarrow (f^{-1})'(2) = 2\left(\dfrac{3}{1}\right)\left(\dfrac{-2}{1^2}\right) = \boxed{-12}$