نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس دوازدهم ریاضی A

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$g'(x)=(2\tan x\times(1+\tan^2 x)-\sqrt{2}\sin x)\times f'(\tan^2 x+\sqrt{2}\cos x)$

$g'(\frac{\pi}{4})=3f'(2)=\sqrt{3}\longrightarrow f'(2)=\frac{\sqrt{3}}{3}$

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$f(x)=\frac{\Lambda+\cos^3 x}{4-\cos^2 x}\longrightarrow f'(x)=\frac{(3\cos^2 x)\times(-\sin x)(4-\cos^2 x)-(\Lambda+\cos^3 x)(-2\cos x)(-\sin x)}{(4-\cos^2 x)^2}$

$g(x)=\frac{2}{2-\cos x}\longrightarrow g'(x)=\frac{-2(\sin x)}{(2-\cos x)^2}$

$f'(\frac{\pi}{6})=\frac{153-128\sqrt{3}}{338}\qquad g'(\frac{\pi}{6})=\frac{76-32\sqrt{3}}{169}=\frac{152-64\sqrt{3}}{338}$

$f'(\frac{\pi}{6})-2g'(\frac{\pi}{6})=\frac{153-128\sqrt{3}}{338}+\frac{-304+128\sqrt{3}}{338}=-\frac{151}{338}$

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$f'_+(\frac{3}{4})=4\sqrt{ax}+(4x-3)\times\frac{a}{2\sqrt{ax}}$

$f'_-(\frac{3}{4})=-4\sqrt{ax}-(4x-3)\times\frac{a}{2\sqrt{ax}}$

$f'_+(\frac{3}{4})-f'_-(\frac{3}{4})=8\sqrt{ax}+(8x-6)\times\frac{a}{2\sqrt{ax}}=4\sqrt{3a}=2\sqrt{6}$

$\sqrt{3a}=\frac{\sqrt{6}}{2}\longrightarrow 3a=\frac{6}{4}\longrightarrow a=\frac{1}{2}$

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$y=-ax+2$

$f(4)=-4a+2$

$f'(4)=-a$

$f(4)+f'(4)=-5a+2=-1\longrightarrow a=\frac{3}{5}$

$f'(4)=-\frac{3}{5}$

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$y=\frac{x+5}{7}$

$\frac{ax-1}{3x+1}=\frac{x+5}{7}\longrightarrow 7ax-7=3x^2+16x+5\longrightarrow 3x^2+(16-7a)x+12=0$

$\Delta=(16-7a)^2-4(3)(12)=0\longrightarrow 49a^2-224a+112=0\longrightarrow\begin{cases}a=\frac{4}{7}\\a=\frac{28}{7}\end{cases}$

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$f(2) = 8 + 2a - b = 0 \rightarrow 2a - b = -8 \rightarrow b = -16$

$f'(2) = 3x^2 + a = 12 + a = 0 \rightarrow a = -12$

$b - a = -16 + 12 = \boxed{-4}$

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$f(x) = \sin^n(x^2) \rightarrow f'(x) = n \sin^{n-1}(x^2) \times \cos x \times 2x$

$\lim_{x \to 0} \frac{f(x) f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^2) \times n \times \sin^{n-1}(x^2) \times \cos x \times 2x}{(1-\cos x)^m}$

$\rightarrow \frac{x^{2n} \times n \times x^{4n-2} \times (1 - \frac{x^2}{2}) \times 2x}{(1 - (1 - \frac{x^2}{2}))^m} = \frac{2 \times n \times x^{4n-1} \times (1 - \frac{x^2}{2})}{(\frac{x^2}{2})^m}$

$2^{m+1} \times n \times x^{4n-2m-1} \times (1 - \frac{x^2}{2}) = 2^{\frac{11}{2}}$

$n \times x^{4n-2m-1} \times (1 - \frac{x^2}{2}) = 2^{\frac{9}{2}-m} \rightarrow \begin{cases} \frac{9}{2} - m = 0 \rightarrow m = \frac{9}{2} \\ 4n - 2m - 1 = 0 \rightarrow n = \frac{10}{2} \end{cases}$ $\quad 2m + n = \boxed{\frac{23}{2}}$

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$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - (\sqrt{x}+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^2} = \frac{-\frac{1}{\sqrt{x}}}{x - 2\sqrt{x} + 1}$

$f''(x) = \frac{-\frac{1}{2}(x)^{-\frac{3}{2}}(x - 2\sqrt{x} + 1) - (-\frac{1}{\sqrt{x}})(1 - \frac{1}{\sqrt{x}})}{(x - 2\sqrt{x} + 1)^2} = \frac{\frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}}{(x - 2\sqrt{x} + 1)^2}$

$f''(2) = \frac{\frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}}}{(2 - 2\sqrt{2})^2} = \frac{\frac{1}{4\sqrt{2}}}{12 - 8\sqrt{2}} = \boxed{\frac{1}{96 - 88\sqrt{2}}}$

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$f(-1) = 8(1-a) = 8 - 8a \qquad \frac{1 - 8 + 8a}{0 + 1} = 8a - 7 = -11 \rightarrow a = -\frac{1}{2}$

$f(0) = 1$

$f(x) = (x^2+1)^3(-\frac{1}{2}x + 1)$

$f'(x) = 3(x^2+1)^2 \times 2x \times (-\frac{1}{2}x + 1) + (x^2+1)^3 \times (-\frac{1}{2})$

$f'(-2a) = f'(1) = \boxed{8}$

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$y = \sin x \cos 2x \rightarrow \sin\frac{\pi}{2}\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2}\cos\pi = -1$ $\qquad \frac{-1-0}{\frac{\pi}{2} - \frac{\pi}{4}} = -\frac{4}{\pi}$

$y = \sin^4 x - \cos^4 x = -\cos 2x \rightarrow -\cos\frac{\pi}{2} = 0$, $-\cos\pi = 1$ $\qquad \frac{1-0}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{4}{\pi}$

$\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{1}$

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